

Lösningsgång

TAOP07 – Optimeringslära grundkurs

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① n fabriker
 K kunder

m produkter

t_{ij} tid, produkt i vid fabrik j

T_j totala antalet timmar för fabrik.

p_{ij} kostnad kronor

c_{ijk} transportkostnad

d_{ik} efterfrågan av kund k av produkt i .

x_{ij} = antal enheter av produkt i som tillverkas vid fabrik j .

y_{ijk} = antal enheter av produkt i som skickas från fabrik j till kund k .

$$\min z = \sum_i^m \sum_j^n p_{ij} \cdot x_{ij} + \sum_i^m \sum_j^n \sum_k^K y_{ijk} \cdot c_{ijk}$$

då

$$\sum_i^m x_{ij} \cdot t_{ij} \leq T_j \quad j = 1, \dots, n$$
$$\sum_k^K y_{ijk} = d_{ik}$$

$$\begin{array}{c} \sum_{k=1}^n \\ y_{ijk} = x_{ij} \\ x_{ij} \geq 0 \end{array}$$

$$y_{ijk} \geq 0, i = 1, \dots, m, j = 1, \dots, n, k = 1, \dots, K$$

$$x_{ij} \geq 0$$

(2)

$$\max z = 6x_1 + c_2 x_2 - 4x_3$$

$$3x_1 + x_2 - x_3 \leq 6$$

$$x_1 + x_2 + 2x_3 \leq 3$$

$$x_1, x_2, x_3 \geq 0$$

$$\Rightarrow \max z = 6x_1 + x_2 - 4x_3$$

$$3x_1 + x_2 - x_3 + s_1 = 6$$

$$x_1 + x_2 + 2x_3 + s_2 = 3$$

$$x_1, x_2, x_3 \geq 0, s_1, s_2 \geq 0$$

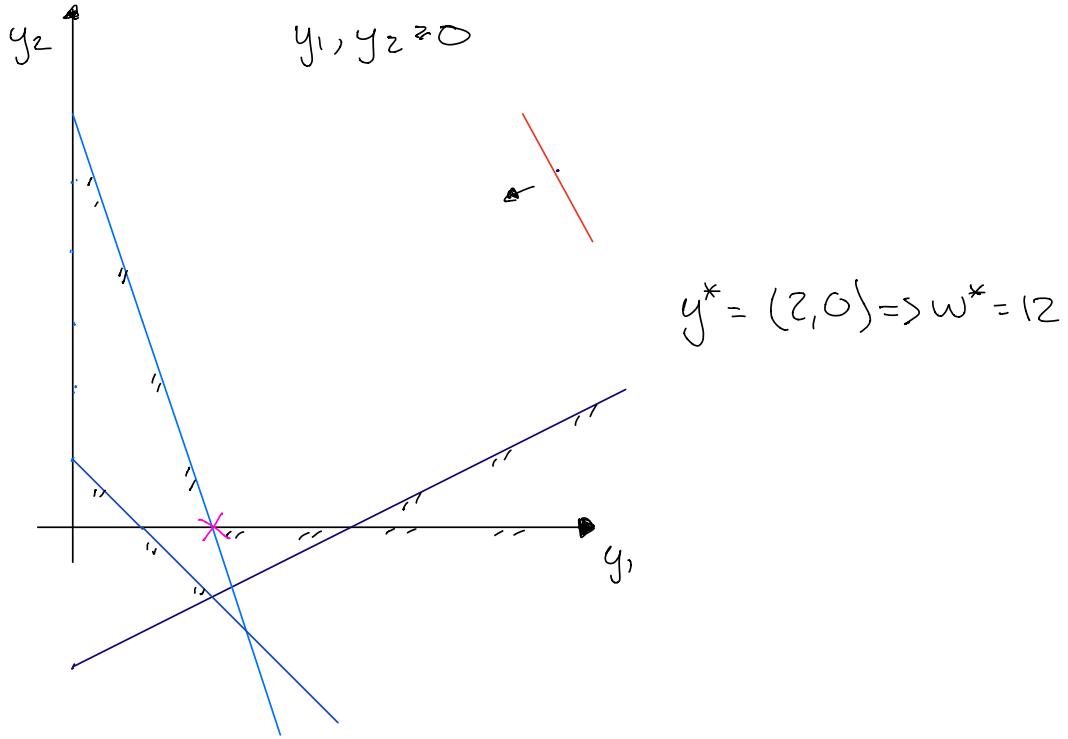
BAS	\downarrow	z	x_1	x_2	x_3	s_1	s_2	Värde
\underline{z}		1	-6	-1	4	0	0	0
$\leftarrow s_1$		0	3	1	-1	1	0	6
s_2		0	1	1	2	0	1	3

Inbemannande: x_1 , utgående s_1

BAS	z	x_1	x_2	x_3	s_1	s_2	Värde
\underline{z}	1	0	1	2	2	0	12
x_1	0	3	1	-1	1	0	6
s_2	0	0	$2/3$	$7/3$	$1/3$	1	1

$$z^* = 12, x^* = (2, 0, 0)$$

$$\begin{aligned}
 b) \quad \min w &= 6y_1 + 3y_2 \\
 3y_1 + y_2 &\geq 6 \\
 y_1 + y_2 &\geq 1 \\
 -y_1 + 2y_2 &\geq -4
 \end{aligned}$$



$$c) \quad \bar{x} = (1, \frac{1}{2}, \frac{1}{2})^\top$$

$$\begin{aligned}
 c^\top \bar{x}^* &= b^\top \bar{y}^\top \Rightarrow \begin{pmatrix} 6, & c_2, & -4 \end{pmatrix} \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \end{pmatrix} = (6, 3) \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = \\
 &= 6 + \frac{c_2}{2} - 2 = 12 \Leftrightarrow \frac{c_2}{2} = 10 \Leftrightarrow \underline{\underline{c_2 = 16}}
 \end{aligned}$$

Komplement sätzen

$$\begin{aligned}
 y_1^* (6 - 3\bar{x}_1^* - \bar{x}_2^* + \bar{x}_3^*) &= 0 \\
 \Rightarrow y_1^* \underbrace{(6 - 3 - \frac{1}{2} + \frac{1}{2})}_{\neq 0} &= 0 \Rightarrow y_1^* = 0
 \end{aligned}$$

$$y_2^* (3 - x_1 - x_2 - 2x_3) = 0 \Rightarrow$$

$$y_2^* (3 - 1 - \frac{1}{2} - 1) = 0 \Rightarrow y_2^* = 0$$

$y_1^*, y_2^* = 0 \Rightarrow$ Finns ej något g som är tillåtet.

$$\begin{array}{ll} \textcircled{3} \quad \max z = x_1 + 2x_2 & \max z = x_1 + 2x_2 + C_2 x_3 \\ \text{dä} \quad -2x_1 + x_2 \leq 2 & \text{dä} \quad -2x_1 + x_2 + x_3 \leq 2 \\ -x_1 + 2x_2 \leq 7 & -x_1 + 2x_2 + x_3 \leq 7 \\ x_1 \leq 3 & x_1 + x_3 \leq 3 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

$$\bar{C}_{N+1} = C_{N+1} - y^{*T} A_{N+1} \Rightarrow$$

$$\begin{aligned} \bar{C}_3 &= C_3 - y^{*T} A_3, \quad y^{*T} = C_B^T B^{-1} = (2, 1, 0) \begin{pmatrix} 1 & -2 & 1 \\ 2 & -1 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \\ &= (2, 1, 0) \begin{pmatrix} 0 & 1/2 & 1/2 \\ 0 & 0 & 1 \\ 1 & -1/2 & 3/2 \end{pmatrix} = (0, 1, 2) \end{aligned}$$

$$\Rightarrow \bar{C}_3 = C_3 - (0, 1, 2) \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = C_3 - 1 + 2 = C_3 - 3 \leq 0$$

$$\Rightarrow C_3 \leq 3$$

$$b) z^* = z^*(b) = C_B^T B^{-1} b = y^{*T} b$$

$$y^{*T} = (0, 1, 2)$$

$$b = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \Rightarrow \underline{\underline{z^* = 17}}$$

$$\begin{aligned} \begin{pmatrix} x_2 \\ x_1 \\ s_1 \end{pmatrix} &= B^{-1} \begin{pmatrix} 2 \\ 7+\Delta \\ 3 \end{pmatrix} = \begin{pmatrix} 0 & 1/2 & 1/2 \\ 0 & 0 & 1 \\ 1 & -1/2 & 3/2 \end{pmatrix} \begin{pmatrix} 2 \\ 7+\Delta \\ 3 \end{pmatrix} + \begin{pmatrix} 0 & 1/2 & 1/2 \\ 0 & 0 & 1 \\ 1 & -1/2 & 3/2 \end{pmatrix} \begin{pmatrix} 0 \\ \Delta \\ 0 \end{pmatrix} = \\ &= \begin{pmatrix} 5 \\ 3 \\ 3 \end{pmatrix} + \begin{pmatrix} 1/2 \\ \Delta \\ -1/2 \end{pmatrix} \Delta \geq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Leftrightarrow -10 \leq \Delta \leq 6 \Rightarrow \Delta = 4 \text{ tillåtet.} \end{aligned}$$

$$④ \min f(x) = x_1^2 + x_2^2 - x_1 x_2 - 3x_2$$

$$\nabla f = (2x_1 - x_2, 2x_2 - x_1, -3)$$

$$x^0 = (-1, 1)^T$$

$$d^k = -\nabla f(x^k) + \beta_{k-1} d^{k-1}, k=1, 2, 3 \dots$$

$$\nabla f(x^0) = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$$

$$x' = x^0 + t, d^0 = / d^0 = -\nabla f(x^0) = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad / = \\ = \begin{pmatrix} -1 \\ 1 \end{pmatrix} + t \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -1+3t \\ 1 \end{pmatrix}$$

$$\min f\left(\begin{pmatrix} -1+3t \\ 1 \end{pmatrix}\right) = (-1+3t)^2 + 1 - (-1+3t) - 3 = \\ = 1 - 6t + 9t^2 + 1 + 1 - 3t - 3$$

$$\Leftrightarrow 9t^2 - 9t \quad \Leftrightarrow 9t(t-1)$$

$$\varphi'(t) = 18t - 9 = 0 \Rightarrow t = \frac{1}{2}$$

$$\varphi''(t) = 18 > 0 \Rightarrow \text{minimum} \Rightarrow x' = \begin{pmatrix} 1/2 \\ 1 \end{pmatrix}$$

2:

$$x' = \begin{pmatrix} 1/2 \\ 1 \end{pmatrix}$$

$$d' = -\nabla f(x') + \beta_0 \cdot d^0$$

$$-\nabla f(x') = \left(0, \frac{3}{2}\right)$$

$$\beta_0 = \frac{\left(\frac{3}{2}\right)^2}{3^2} = \frac{1}{4}$$

$$\Rightarrow d' = \left(0, \frac{3}{2}\right) + \frac{1}{4} \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \left(\frac{3}{4}, \frac{3}{2}\right)$$

$$x^2 = x^1 + t_1 d^1 = \begin{pmatrix} 1/2 \\ 1 \end{pmatrix} + t_1 \begin{pmatrix} \frac{3}{4}, \frac{3}{2} \end{pmatrix}^\top = \begin{pmatrix} 1/2 + t_1 \frac{3}{4} \\ 1 + t_1 \frac{3}{2} \end{pmatrix}$$

$$\text{min } f\left(\begin{pmatrix} 1/2 + t_1 \frac{3}{4} \\ 1 + t_1 \frac{3}{2} \end{pmatrix}\right) = \left(\frac{1}{2} + t_1 \frac{3}{4}\right)^2 + \left(1 + t_1 \frac{3}{2}\right)^2 -$$

$$- \left(\frac{1}{2} + t_1 \frac{3}{4}\right)\left(1 + t_1 \frac{3}{2}\right) - 3 - t_1 \frac{9}{2} =$$

$$= \frac{1}{4} + \frac{3t_1}{4} + \frac{9t_1^2}{16} + 1 + \frac{6t_1}{2} + t_1^2 \frac{9}{4} - \frac{1}{2} - t_1 \frac{6}{4} - \frac{9t_1^2}{8} - 3$$

$$- t_1 \frac{9}{2} = \varphi(t)$$

$$\varphi(t) = \frac{3}{4} + \frac{9t_1}{8} + \frac{6}{2} + \frac{18}{4}t_1 - \frac{6}{4} - \frac{9t_1}{4} - \frac{9}{2} =$$

$$= \frac{3}{4} + \frac{12}{4} - \frac{6}{4} - \frac{18}{4} + \frac{9t_1}{8} + \frac{36t_1}{8} - \frac{18t_1}{8} =$$

$$= -\frac{9}{4} + \frac{27t_1}{8} = 0 \Leftrightarrow t_1 = \frac{9 \cdot 8}{4 \cdot 27} = \frac{72}{108} = t_1 = \frac{2}{3}$$

$$\Rightarrow x^2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$\nabla f(x^2) = (0, 0)$ stationär punkt

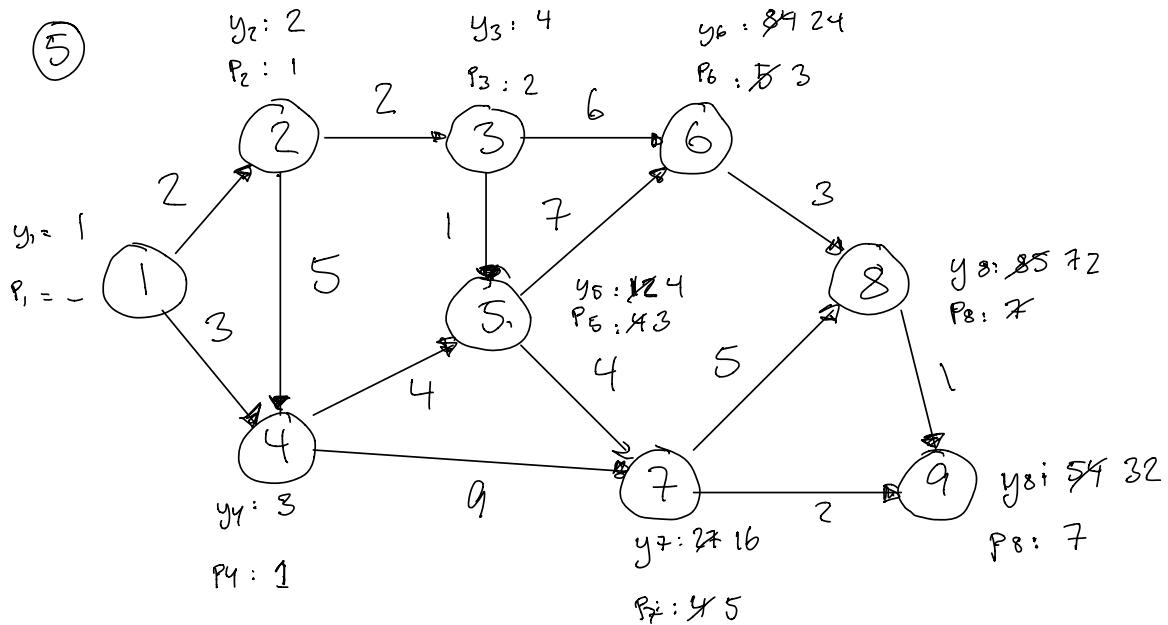
$$\nabla^2 f(x) = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \quad \det(\nabla^2 f(x) - \lambda I) = \begin{vmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} =$$

$$= (2-\lambda)(2-\lambda) - 1 = 4 - 4\lambda + \lambda^2 - 1 = 0$$

$$\Leftrightarrow (\lambda - 2)^2 - 4 + 3 = 0 \Leftrightarrow \lambda = 2 \pm 1 \geq 0$$

\Rightarrow positiv definit \Rightarrow minimum $\Rightarrow f(x)$ hat ein Minimum in \mathbb{R}^2

(5)



$$BV: 1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 7 \rightarrow 9 \quad 50+35$$

$$\Rightarrow 1 - 0,98 \times 0,98 \times 0,99 \times 0,96 + 0,98 \approx 0,7.$$

(6)

$$Z^* = \min 25x_1 + 17x_2 + 28x_3 + 19x_4$$

$$\text{d} \ddot{\text{a}} \quad 6x_1 + 4x_2 + 7x_3 + 5x_4$$

$$x = (0, 1, 0, 1)$$

$$x = (1, 0, 0, 1)$$

$$x = (1, 0, 1, 0)$$

$$x = (0, 0, 1, 1)$$

$$Z^* = \min 25x_1 + 17x_2 + 28x_3 + 19x_4 + \sqrt{(11 - 6x_1 - 4x_2 - 7x_3 - 5x_4)}$$

$$\text{d} \ddot{\text{a}} \quad x = (0, 1, 0, 1)$$

$$x = (1, 0, 0, 1)$$

$$x = (1, 0, 1, 0)$$

$$x = (0, 0, 1, 1)$$

$$h(v) =$$

$$\Rightarrow z^* = 11v + \min(25 - 6v)x_1 + (17 - 4v)x_2 + (28 - 7v)x_3 \\ + (19 - 5v)x_4$$

$$h(3) = 33 + \min(7x_1 + 5x_2 + 7x_3 + 4x_4) = 33 + 6 \Rightarrow \text{optimalen.}$$

$$h(4) = 44 - 1 = 43 \text{ optimalen! } x = (0, 0, 1, 1) \Rightarrow z^* = 47$$

$$\Rightarrow 43 \leq z^* \leq 47$$

(7)

b)

