

$$[6.36] \text{ Låt } n = 2^{12} \cdot 3^8 \cdot 5^6 \cdot 7^2 \cdot 11^4 \cdot 13^3 \cdot 23 \cdot 151$$

a) Antal delare (pos och neg): exponenterna +1,
varför +9?

$$2 \cdot (13 \cdot 9 \cdot 7 \cdot 3 \cdot 5 \cdot 4 \cdot 2 \cdot 2) = 393120 \quad \checkmark$$

b) Pos. delare: $196560 = 13 \cdot 9 \cdot 7 \cdot 3 \cdot 5 \cdot 4 \cdot 2 \cdot 2$

c) $7 \cdot 5 \cdot 4 \cdot 2 \cdot 3 \cdot 2 = 1680$

d) $8 \cdot 6 \cdot 3 \cdot 1$

e) $3 \cdot 4 \cdot 2 \cdot 2 \cdot 3$ varför 3^2

[8.3]

a) $5^{111} \pmod{12}$

$$5^2 = 25 \equiv 12 \cdot 2 + 1 \equiv 1 \pmod{12}$$

$$5^4 = (5^2)^2 \equiv 25^2 \equiv 1^2 = 1$$

$$5^{1000} = (5^2)^{500} \equiv 25^{500} \equiv 1^{500}$$

$$5^{100} = (5^2)^{50} \equiv 25^{50} \equiv 1^{50}$$

$$5^{10} = (5^2)^5 \equiv 25^5 \equiv 1^5$$

$$5^1 = 5$$

Stu-klassen $\pmod{12}$

$$\{0\} : \{ \dots, -12, 0, 12, 24, \dots \}$$

$$\{1\} : \{ \dots, -11, 1, 13, 25, \dots \}$$

$$\{2\} : \{ \dots, -10, 2, 14, 26, \dots \}$$

$$\{3\} : \{ \dots, -9, 3, 15, 27, \dots \}$$

$$\{4\} : \{ \dots, -8, 4, 16, 28, \dots \}$$

$$\{5\} : \{ \dots, -7, 5, 17, 29, \dots \}$$

$$5^{111} = 5^{1000} \cdot 5^{100} \cdot 5^{10} \cdot 5^1 \equiv 1 \cdot 1 \cdot 1 \cdot 5 = 5$$

Svar: 5

b) $3^{41} \pmod{79}$

$$3^4 = 81 \equiv 2 \pmod{79}$$

$$3^8 = (3^4)^2 = (9^2)^2 \equiv 2^2 = 4 \pmod{79}$$

$$3^{40} = (3^8)^5 \equiv 4^5 = 1024 = 79 \cdot 12 + 76 \equiv 76$$

$$3^{41} = 3^{40} \cdot 3^1 \equiv 76 \cdot 3 = 228 \equiv 79; 2 + 70 \equiv 70$$

Svar: 70

c) $4^{220} \pmod{19}$

$$4^2 = 16 \equiv 16$$

$$4^4 = (4^2)^2 \equiv 16^2 = 256 = 19 \cdot 13 + 9 \equiv 9$$

$$4^8 = (4^4)^2 \equiv 9^2 = 81 = 19 \cdot 4 + 5 \equiv 5$$

$$4^{16} = (4^8)^2 \equiv 5^2 = 25 = 1 \cdot 19 + 6 \equiv 6$$

$$4^{32} = (4^{16})^2 \equiv 6^2 = 36 = 1 \cdot 19 + 17 \equiv 17$$

$$4^{64} = (4^{32})^2 \equiv 17^2 = 289 \equiv 4$$

$$4^{220} = \underbrace{4^{64} \cdot 4^{64} \cdot 4^{64}}_{\cdot 4^4 \cdot 4^4 \cdot 4^4} \cdot 4^8 \cdot 4^4$$

$$\equiv 4^3 \cdot 5 \cdot 9^5$$

[6.15]

$$36x + 51y = 21 \Leftrightarrow 12x + 17y = 7 \quad \text{osv...}$$

(i statistikhäfte)

[6.18] $\Rightarrow 14x + 165y = c$

a) $84x + 990y = c \quad 10 < c < 20$

$$990 = 11 \cdot 84 + 66$$

$$84 = 1 \cdot 66 + 18$$

$$66 = 3 \cdot 18 + 12$$

$$18 = 1 \cdot 12 + 6$$

$$\therefore \text{sgd}(84, 990) = 6$$

Eku har lösningar om

$$6 | c \Rightarrow$$

saknar lösning om $c \neq 12, 18$

b) om $c=12 \Rightarrow$

$$6 = 18 - 1 \cdot 12 = 18 - 1(66 - 3 \cdot 18) = 4 \cdot 18 - 1 \cdot 66 =$$

$$4(84 - 1 \cdot 66) - 1 \cdot 66 = 4 \cdot 84 - 5 \cdot 66 = 4 \cdot 84 - 5(990 - 11 \cdot 84) =$$

$53 \cdot 84 - 5 \cdot 990$, detta ger att

$$12 = 118 \cdot 84 - 10 \cdot 990 \Rightarrow \text{svar: } x = 118 - 165n$$

$$y = -10 + 14n$$

Fråga!



Om man inte förkortar så långt som möjligt i första ledet, missar man lösningar
Vä?

[6.19]

$$\begin{cases} x+y+z = 50 & (1) \\ 13x+31y+z = 116 & (2) \end{cases} \Rightarrow$$

lösning om $\underbrace{\text{sgd}(1,1,1)}_{=1} | 50$ och $\underbrace{\text{sgd}(13,31,1)}_{=1} | 116$

(1)

$\text{sgd}(1,1)=1 \Rightarrow$ hjälpekv: $x+n=50$ med lösningar:

$$\begin{cases} x = 25 - 1n \\ n = 25 + 1n \end{cases}$$

Nyt försök:

$$(2) - (1) \Rightarrow$$

$$12x+30y = 66 \Rightarrow 4x+10y = 22 \Rightarrow 2x+5y = 11$$

$$5 = 2 \cdot 2 + 1 \Rightarrow 1 = 5 - 2 \cdot 2 \Rightarrow 11 = 11 \cdot 5 - 22 \cdot 2 \Rightarrow$$

$$\begin{cases} x = -22 + 15n \\ y = 11 - 8n \end{cases}$$

$$(1) \text{ ger } z = 50 - (-22 + 5n) - (11 - 2n) = 61 - 3n$$

$$\begin{cases} x = -22 + 5n \\ y = 11 - 8n \\ z = 61 - 3n \end{cases}$$

b) om $n=5 \Rightarrow (3, 1, 46)$

c) n måste minst vara 1a, funkar det?

Japp: $(33, -11, 28)$

$(38, -13, 25)$

$(43, -15, 22)$

{6.33]

a) $2x_1 + 3x_2 + 4x_3 = 5$ $x_3 = t$ $2x_1 + 3x_2 = 5 - 4t$
 $\text{sgd}(2,3) = 1$ $| \quad 5 = 4t$

$\text{sgd}(3,4) = 1 \Rightarrow \text{Hjälpduk: } 2x_1 + y = 5$

$2 = 2 \cdot 1$

$$\begin{cases} x_1 = 2 + n \\ y = 3 - 2n \end{cases}$$

$3x_2 + 4x_3 = 3 - 2n$

med lösningssät: $x_2 = 3 - 2n$

$x_2 = -2 + 2n$

$\text{sgd}(2,3) = 1 \Rightarrow 2y + 4x_2 = 5$

$y = 1 \rightarrow y =$

$x_3 =$

[6.84]

$$\begin{array}{c} \left[\begin{array}{l} x+y = -3 \\ x-y = 1 \end{array} \right] \quad \left[\begin{array}{l} x+y = 1 \\ x-y = -3 \end{array} \right] \quad \left[\begin{array}{l} x+y = 1 \\ x-y = 3 \end{array} \right] \quad \left[\begin{array}{l} x+y = 1 \\ x-y = 1 \end{array} \right] \\ \text{---} \\ \left(\begin{array}{l} x+y \\ x-y \end{array} \right) \end{array}$$

Tus dessa

$$\Rightarrow (x, y) = (\pm 2, \pm 1) \quad (\text{alla kombinationer})$$

$$[8.4] \quad 8 | 3^{2n} - 1$$

① Bas:

$$3^{2 \cdot 1} - 1 = 9 - 1 = 8 \Rightarrow 8 | 8 \quad \text{OK!}$$

② Antagande:

$$3^{2p} - 1 = 8m, m \text{ heltal} > 0$$

③ Steg:

$$3^{2(p+1)} - 1 = 3^{2p+2} - 1 = 3^{2p} \cdot 3^2 - 1 = \{ \text{(2) ger} \}$$

$$\begin{aligned} 3^{2p} &= 8m + 1 \\ 3^{2p} &= (8m + 1) 3^2 - 1 = 8m + 9 - 1 = 8m + 8 = \\ 8(m+1) &\Rightarrow 8 | 3^{2(p+1)} - 1 \end{aligned}$$

Enl. induktionsprincipen visar detta att $8 | 3^{2n} - 1$
 $n \in \mathbb{N}$

$$[8.5] \quad a = a_k \cdot 10^k + a_{k-1} \cdot 10^{k-1} + \dots + a_1 \cdot 10 + a_0$$

$$a) \quad 10 \equiv 0 \pmod{2} \Rightarrow 10^k \equiv 0 \pmod{2} \Rightarrow$$

$a = a_k \cdot 10^k + \dots + a_0 \equiv a_0 \pmod{2}$. Alltså är a delbart med 2 om och endast a_0 .

$$b) \quad 10 \equiv 2 \pmod{4} \Rightarrow 10^k \equiv 2 \pmod{4} \quad \{ 10^k = (2 \cdot 5)^k = 2^k \cdot 5^k \}$$

$$\begin{aligned} a &= 2^k \cdot 5^k a_k + 2^{k-1} \cdot 5^{k-1} a_{k-1} + \dots + 10a_1 + a_0 = \\ &= \underbrace{2^2 \cdot 2^{k-2} \cdot 5^k a_k + 2^2 \cdot 2^{k-3} \cdot 5^{k-1} a_{k-1} + \dots + 2^2 \cdot 5^2 a_2}_{\text{delbart med } 2^2 = 4} + 10a_1 + a_0 \end{aligned}$$

$\therefore 4 | a \text{ om och endast } 4 | 10a_1 + a_0$

$$d) \quad 10 \equiv 4 \pmod{6} \Rightarrow 10^7 \equiv 4 \pmod{6}$$

$$a = 10^k a_k + \dots + 10a_1 + a_0 =$$

eftersom $\text{mgm}(2, 3) = 6$

[8.14]

$$7^{42} = (7^2)^{21} = 49^{21} \equiv 49 \pmod{100}$$

Svar: 49

[8.15]

$$2^{1257} = 2^{1000} \cdot 2^{200} \cdot 2^{80} \cdot 2^7 \quad (1)$$

$$2^2 \equiv 4 \pmod{10}$$

$$2^4 = (2^2)^2 \equiv 4^2 = 16 \equiv 6 \pmod{10}$$

$$2^8 = (2^4)^2 \equiv 6^2 = 36 \equiv 6 \pmod{10}$$

$$2^{16} = (2^8)^2 \equiv 6^2 = 36 \equiv 6 \pmod{10}$$

$$2^{20} = 2^{16} \cdot 2^{16} \cdot 2^{16} \cdot 2^2 \equiv 6 \cdot 6 \cdot 6 \cdot 4 = 864 \equiv 4$$

$$2^{200} = 2^8 \cdot 2^{80} \cdot 2^{80} \cdot 2^{80} \equiv 2 \cdot 2 \cdot 2 \cdot 2 = 16 \equiv 6 \pmod{10}$$

$$2^{1000} = 2^{200} \cdot 2^{200} \cdots 2^{200} \equiv 6^5 \equiv 6 \pmod{10}$$

$$\Rightarrow (1) \equiv 6 \cdot 6 \cdot 4 \cdot 6 \cdot 4 \cdot 2 = 6912 \equiv 2$$

[SVAR: 2]

