Uppgifter 4

TAOP07 – Optimeringslära grundkurs

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Lektionsgenomgång

$$x_B = B^{-1} b \geq 0$$

$$\bar{c}_N^T = c^T N - \underbrace{c_B^T B^{-1}}_{v} N \quad \begin{matrix} \geq 0 & \text{min} \\ \leq 0 & \text{max} \end{matrix}$$

$$z(x_n) = c_B^T \underbrace{B^{-1} b}_{} + \bar{c}_N^T x_N$$

x_B computed for $x_N = 0$

$$\min z = 2x_1 - 3x_2 + 4x_3$$

då

(P)

$$\begin{aligned} x_1 + x_2 + x_3 &\geq 1 && v_1 \\ 6x_1 + 5x_2 - 4x_3 &\leq 4 && v_2 \\ -7x_1 + 8x_2 + 9x_3 &= 3 && v_3 \end{aligned}$$

$$x_1 \geq 0, \quad x_2 \leq 0, \quad x_3 \text{ fri}$$

$$\max w = 1 \cdot v_1 + 4v_2 + 3v_3$$

(D)

$$\begin{aligned} 1 \cdot v_1 + 6v_2 - 7v_3 &\leq 2 && \leftarrow x_1 \\ 1 \cdot v_1 + 5v_2 + 8v_3 &\geq -3 && \leftarrow x_2 \\ -7v_1 + 8v_2 + 9v_3 &= 4 && \leftarrow x_3 \end{aligned}$$

$$v_1 \geq 0 \quad v_2 \leq 0 \quad v_3 \text{ fri}$$

53

$$z^* = \max z = \overbrace{400x_1 + 200x_2 + 250x_3}^{c_B}$$

da

$$\begin{array}{rcrcrcl|l} 6x_1 & + & 2x_2 & + & 3x_3 & \le & 2000 & y_1 \\ 8x_1 & + & 2x_2 & + & 3x_3 & \le & 3000 & y_2 \\ x_1 & + & x_2 & + & x_3 & \le & 625 & y_3 \end{array}$$

$$x_1, x_2, x_3 \ge 0$$

$$w^* = \min w = 2000y_1 + 3000y_2 + 625y_3$$

da

$$(D) \quad \begin{array}{rcrcrcl|l} 6y_1 & + & 8y_2 & + & y_3 & \ge & 400 & x_1 \\ 2y_1 & + & 2y_2 & + & y_3 & \ge & 200 & x_2 \\ 3y_1 & + & 3y_2 & + & y_3 & \ge & 250 & x_3 \end{array}$$

$$y_1, y_2, y_3 \ge 0$$

Komplementvinkor

$$x_1^* (6y_1^* + 8y_2^* + y_3^* - 400) = 0$$

$$\underset{x_1^* > 0}{\Rightarrow} \quad 6y_1^* + 8y_2^* + y_3^* = 400$$

$$x_2^* (2y_1^* + 2y_2^* + y_3^* - 200) = 0$$

$$\Rightarrow \quad 2y_1^* + 2y_2^* + y_3^* = 200$$

$$x_3^* (3y_1^* + 3y_2^* + y_3^* - 250) = 0$$

$\Rightarrow$ inget, ty $x_3^* = 0$

$$y_1^* (-6x_1^* - 2x_2^* - 3x_3^* + 2000) = 0$$

$$y_2^* (-8x_1^* - 2x_2^* - 3x_3^* + 3000) = 0$$

$$\Rightarrow \quad y_2^* = 0$$

$$y_3^*(-x_1^* - x_2^* - x_3^* + 625) = 0$$

$$\begin{aligned} 6y_1^* + y_3^* &= 400 \\ 2y_1^* + y_3^* &= 200 \end{aligned} \Rightarrow \left(\begin{array}{cc|c} 6 & 1 & 400 \\ 2 & 1 & 200 \end{array}\right) \uparrow -3 \sim \left(\begin{array}{cc|c} 0 & -2 & -200 \\ 2 & 1 & 200 \end{array}\right)$$

$$y_3^* = 100$$

$$y_1^* = \frac{200 - 100}{2} = \frac{100}{2} = 50,$$

$$\underline{y_2^* = 0}$$

(47)

$$z^* = \min z = \quad -x_2 - x_3 + 2x_4$$

$$\text{dá} \quad \begin{array}{l} x_1 + x_2 + x_3 + x_4 = 0 \\ -x_1 + x_2 \qquad\quad + x_4 = 1 \end{array} \left| \begin{array}{l} y_1 \\ y_2 \end{array} \right.$$

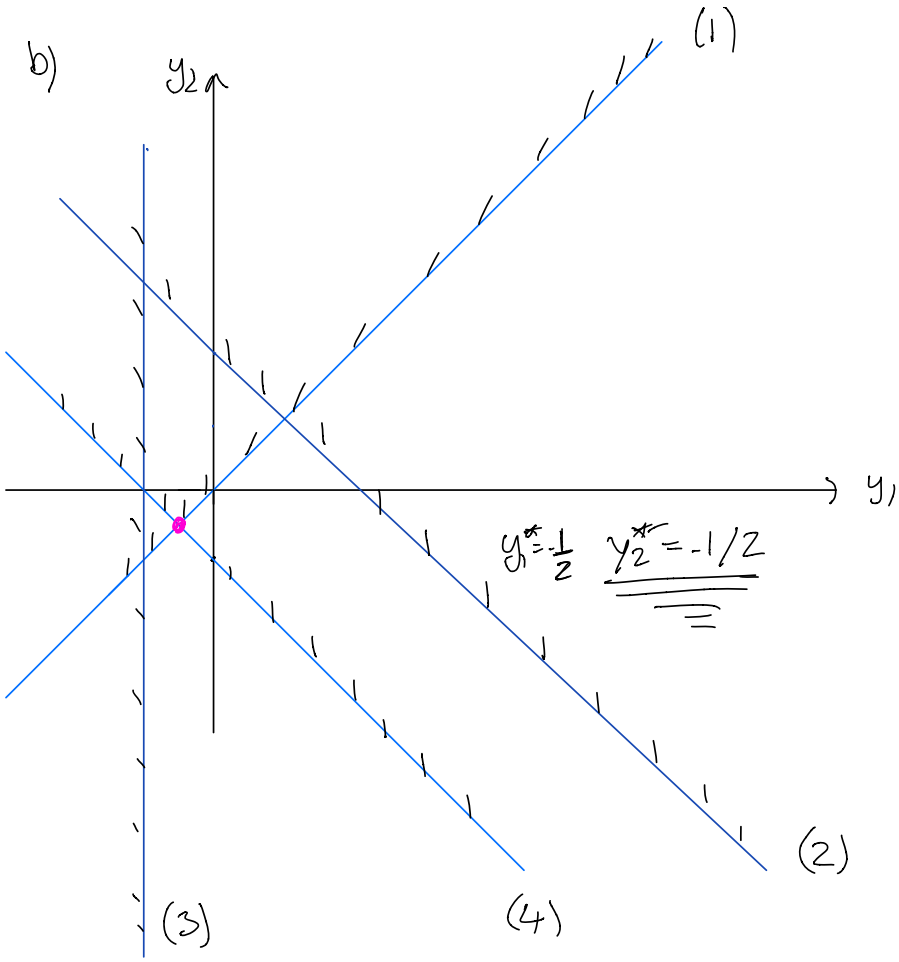
$$(P) \quad x_1, x_3 \leq 0, \; x_2, x_4 \geq 0$$

$$w^* = \max w = \quad y_2$$

$$(D) \quad \begin{array}{lll} y_1 - y_2 \geq 0 & (1) & \\ y_1 + y_2 \leq -1 & (2) & y_2 \leq -1 - y_1 \\ y_1 \geq -1 & (3) & \\ y_1 + y_2 \leq 2 & (4) & \end{array}$$

y_1, y_2 fria

b)



$y_1^* = -\frac{1}{2}$ $\quad y_2^* = -1/2$

$$\begin{aligned} y_1 - y_2 &\geq 0 \\ y_1 + y_2 &\leq -1 \\ y_1 &\geq -1 \\ y_1 + y_2 &\leq 2 \end{aligned}$$

c) Komplementvillkor

$$\begin{aligned} x_1^*(y_1^* - y_2^*) &= 0 \Rightarrow x_1^*\overbrace{\left(-\tfrac{1}{2} + \tfrac{1}{2}\right)}^{=0} = 0 \\ x_2^*(y_1^* + y_2^* + 1) &= 0 \qquad x_2^*\underbrace{\left(-\tfrac{1}{2} - \tfrac{1}{2} + 1\right)}_{=0} = 0 \\ x_3^*(y_1^* + 1) &= 0 \qquad x_3^* = 0 \\ x_4^*(y_1^* + y_2^* - 2) &= 0 \qquad x_4^* = 0 \end{aligned}$$

$$\begin{aligned} y_1 &\leq 1 \\ y_1 &\geq 1 \end{aligned}$$

$$\begin{aligned} y_1^*(-x_1 - x_2 - x_3 - x_4) &= 0 \Rightarrow x_1^* + x_2^* + x_3^* + x_4^* = 0 \\ y_2^*(+x_1 - x_2 - x_4 - 1) &= 0 \qquad -x_1^* + x_2^* + x_4^* + 1 = 0 \end{aligned}$$

$$\begin{aligned} x_1^* + x_2^* &= 0 \\ -x_1^* + x_2^* - 1 &= 0 \end{aligned} \Rightarrow \left(\begin{array}{cc|c} 1 & 1 & 0 \\ -1 & 1 & 1 \end{array}\right) \approx \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 2 & 1 \end{array}\right) = \begin{aligned} x_1^* &= 0 - x_2^* = -\tfrac{1}{2} \\ x_2^* &= \tfrac{1}{2} \end{aligned}$$

Stoppa in den!

(48) $z^* = \min\ 3x_1 + 2x_2 + 2x_3$

$$X_B = \begin{pmatrix} x_2 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 2 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 8 \\ 1 \end{pmatrix}$$