

$$R_n = R_{n-1} + n$$

$$R_n = (R_{n-2} + (n-1)) + n = R_{n-2} + (n-1) + n$$

$$\vdots = (R_{n-3} + (n-2)) + (n-1) + n = \dots$$

$$\vdots = R_{n-4} + (n-3) + (n-2) + (n-1) + n.$$

\vdots

$$= R_3 + 4 + \dots + (n-3) + (n-2) + (n-1) + n$$

$$R_2 + 3 + 4 + \dots + (n-2) + (n-1) + n$$

$$R_1 + 2 + 3 + 4 + \dots + n$$

$$= R_0 + 1 + 2 + \dots + n = 1 + \{1 + 2 + 3 + \dots + n\} =$$

$\underbrace{\hspace{1cm}}_{=1}$

$$= 1 + \underbrace{\frac{n(1+n)}{2}}$$

svær

Lektion 6

3

$$c) a_n - 5a_{n-1} + 6a_{n-2} = 0 \quad a_0 = 1, a_1 = 0$$

Kar. eku:

$$r^2 - 5r + 6 = 0 \Rightarrow \left\{ r = \frac{5}{2} + \alpha \sqrt{\frac{25}{4} - \frac{24}{4}} = \frac{5}{2} + \frac{1}{2} = 3, 2 \right\}$$

$(r-3)(r-2)=0$, dette gir:

$$a_n = \alpha 3^n + \beta \cdot 2^n \quad \text{BV ger} \begin{cases} 1 = \alpha + \beta \\ 0 = 3\alpha + 2\beta \end{cases} \quad \begin{cases} 1 = \alpha + \beta \\ -2 = \alpha \end{cases} \quad \begin{cases} \alpha = -2 \\ \beta = 3 \end{cases}$$

$$\therefore a_n = -2 \cdot 3^n + 3 \cdot 2^n \quad R$$

$$d) a_n - 4a_{n-1} + 4a_{n-2} = 0 \quad a_0 = 6, a_1 = 8$$

$$r^2 - 4r + 4 = 0 \quad \left\{ r = 2 \pm \sqrt{4-4} \right\} \Rightarrow$$

$$a_n = \alpha \cdot 2^n + \beta \cdot n \cdot 2^n \quad \text{BV:} \quad \begin{cases} 6 = \alpha \\ 8 = 2\alpha + 2\beta \end{cases} \quad \begin{cases} \alpha = 6 \\ \beta = -2 \end{cases}$$

$$\Rightarrow a_n = 6 \cdot 2^n - 2 \cdot n \cdot 2^n \quad R$$

$$e) a_n + 4a_{n-1} + 4a_{n-2} = 0 \quad a_0 = 0, a_1 = 1$$

$$r^2 + 4r + 4 = 0 \quad \left\{ r = -2 \pm \sqrt{4-4} \right\}$$

$$a_n = \alpha (-2)^n + \beta n (-2)^n$$

BV:

$$\begin{cases} 0 = \alpha \\ 1 = \alpha + 2\beta \end{cases} \quad \begin{cases} \alpha = 0 \\ \beta = -1/2 \end{cases}$$

$$\therefore a_n = -\frac{1}{2} \cdot n (-2)^n = n \left(-\frac{2}{2} \right)^{n-1}$$

[47]

$$a_n = 5a_{n-2} - 4a_{n-4}, \quad a_0 = 3, \quad a_1 = 2, \quad a_2 = 6, \quad a_3 = 8$$

$$a_n - 5a_{n-2} + 4a_{n-4} = 0$$

$$r^4 - 5r^2 + 4 = 0 \Rightarrow (t = r^2) \Rightarrow t^2 - 5t + 4 = 0 \Rightarrow t = \frac{5}{2} \pm \sqrt{\frac{25-16}{4}}$$

$$\Rightarrow t = \frac{5}{2} \pm \frac{3}{2} = 4, 1, \text{ wobei } gcr \dots =$$

$$r^2 = t \Rightarrow r = \pm \sqrt{t} = -4, 4, -1, 1, \text{ vi für}$$

$$a_n = \alpha \cdot 4^n + \beta (-4)^n + \gamma - c$$

$$\left\{ \begin{array}{l} \text{1. G} \\ \text{2. G} \\ \text{3. G} \\ \text{4. G} \end{array} \right\} \left\{ \begin{array}{l} a_0 = \alpha + \beta + \gamma - c = 3 \\ a_1 = 4\alpha - 4\beta + \gamma - c = 2 \\ a_2 = 16\alpha + 16\beta + \gamma - c = 6 \\ a_3 = 64\alpha - 64\beta + \gamma - c = 8 \end{array} \right\} \left\{ \begin{array}{l} \text{1. G} \\ \text{2. G} \\ \text{3. G} \end{array} \right\} \left\{ \begin{array}{l} \alpha + \beta + c = 3 \\ 4\alpha - 4\beta + c = 2 \\ 16\alpha + 16\beta + c = 6 \end{array} \right\}$$

$$\alpha + \beta + c = 3$$

$$-8\beta - 3c = -10 \Rightarrow \beta = \frac{(10+3+14)}{8} = 31/20$$

$$-15c = -42 \Rightarrow c = \frac{14}{5}$$

$$[19] \quad a_n + 3a_{n-1} + 3a_{n-2} + a_{n-3} = 0 \quad a_0 = 5, \quad a_1 = -9, \quad a_2 = 15$$

Kar.ekv.: $r^3 + 3r^2 + 3r + 1 = 0 \quad (-1)$ är en rot!

$$\begin{array}{c} r^3 + 3r^2 + 3r + 1 \\ \hline r^3 + 3r^2 + 3r + 1 \quad | \quad r+1 \\ -(r^3 + r^2) \\ \hline -2r^2 - 2r \\ -(-2r^2 - 2r) \\ \hline r+1 \\ -(r+1) \\ \hline 0 \end{array}$$

$\Rightarrow (r+1)(r^2 + 2r + 1) = (r+1)^3 \quad (-1)$ är alltså en trippelrot!

$$a_n = \alpha_1 (-1)^n + \alpha_2 n (-1)^n + \alpha_3 n^2 (-1)^n$$

$$\left\{ \begin{array}{l} \alpha_0 = 5 = \alpha_1 \\ \alpha_1 = -9 = -\alpha_1 - \alpha_2 - \alpha_3 \\ \alpha_2 = 15 = \alpha_1 + 2\alpha_2 + 4\alpha_3 \end{array} \right. \quad \text{②} \quad \left\{ \begin{array}{l} -9 = -5 - \alpha_2 - \alpha_3 \\ 15 = 5 + 2\alpha_2 + 4\alpha_3 \end{array} \right. \quad \left\{ \begin{array}{l} -3 = -5 + 2\alpha_3 \\ \alpha_2 = 3 \\ \alpha_3 = 1 \end{array} \right. \Rightarrow$$

svar: $a_n = (n^2 + 3n + 5)(-1)^n$

$$[27] \quad a_n = 8a_{n-2} - 16a_{n-4} + F(n)$$

a) $F(n) = n^3$

$$a_n^P = c_0 + c_1 n + c_2 n^2 + c_3 n^3$$

b) $F(n) = (-2)^n$

$$a_n^P = A(-2)^n \cdot n^2 \quad (\text{ty } -2 \text{ är en dubbelrot?})$$

$$\begin{array}{ll} \text{o) } F(n) = n2^n & \text{g) } F(n) = 2 \\ & \Rightarrow a_n^P = P_0 \\ & n^2(P_0 + P_1 n) 2^n \\ & \uparrow \\ & \text{ty dässel} \end{array}$$

$$[29] \quad a_n = 2a_{n-1} + 3^n \quad (1)$$

a) Homogen:

$$a_n - 2a_{n-1} = 0 \Rightarrow r - 2 = 0 \Rightarrow r = 2$$

$$a_n^h = \alpha 2^n$$

$$a_n^p = c 3^n \quad i \quad (1) \text{ ger:}$$

$$c \cdot 3^n = 2c \cdot 3^{n-1} + 3^n \Rightarrow \left\{ \begin{array}{l} \text{dividera med } 3^n \\ \end{array} \right\} \Rightarrow$$

$$c = 2c \cdot 3^{-1} + 1 \Rightarrow c - \frac{2}{3}c = 1 \Rightarrow \frac{1}{3}c = 1 \Rightarrow c = 3$$

$$\Rightarrow a_n^p = 3^{n+1}$$

$$\therefore a_n = \underline{\alpha 2^n + 3^{n+1}}$$

b) $a_1 = 5$ ger

$$5 = 2 \cdot \alpha + g \Rightarrow \alpha = -2$$

$$\therefore a_n = -2 \cdot 2^n + 3^{n+1}$$

$$[28] \quad a_n = 2a_{n-1} + 2n^2$$

Hom:

$$r - 2 = 0 \Rightarrow r = 2 \Rightarrow a_n^h = \alpha \cdot 2^n$$

Part:

$$\text{Ansätt } p_0 + p_1 n + p_2 n^2$$

$$p_0 + p_1 n + p_2 n^2 = 2(p_0 + p_1(n-1) + p_2(n-1)^2) + 2n^2 \Rightarrow$$

$$p_0 + p_1 n + p_2 n^2 = \underline{2p_0 + 2p_1 n - 2p_2} + \underline{2p_2 n^2} - 4p_2 n + 2p_2 + 2n^2$$

$$[317] \quad a_n = 5a_{n-1} - 6a_{n-2} + 2^n + 3n$$

$$a_n - 5a_{n-1} + 6a_{n-2} = 2^n + 3n \quad (1)$$

charakteristisk ekv:

$$r^2 - 5r + 6 = 0 \Rightarrow r = \frac{5}{2} \pm \sqrt{\frac{25}{4} - \frac{24}{4}} = \frac{5}{2} \pm \frac{1}{2} \Rightarrow r = 3, 2$$

Hom. lösning:

$$a_n^h = \alpha_1 3^n + \alpha_2 2^n$$

Partikulär:

$$(P1): \text{ ansatz: } a_n^{p1} = c 2^n$$

$$n c 2^n - 5(n-1) c 2^{n-1} + 6(n-2) c 2^{n-2} = 2^n \Rightarrow$$

$$nc 2^n - 5(n-1)c 2^{n-1} + 6(n-2)c 2^{n-2} = 4 \Rightarrow$$

$$4nc - 10nc + 10c + 6nc - 12c = 4 \Rightarrow$$

$$-2c = 4 \Rightarrow c = -2 \text{ sa}$$

$$a_n^{p1} = -2 \cdot 2^n \cdot n = -n \cdot 2^{n+1}$$

$$(P2): \text{ ansatz: } a_n + b, \text{ i (1) ger:}$$

$$an + b - 5(a(n-1) + b) + 6(a(n-2) + b) = 3n \Rightarrow$$

$$an + b - 5an + 5a - 5b + 6an - 12a + 6b = 3n \Rightarrow$$

$$\begin{cases} 2an = 3n \Rightarrow a = 3/2 \\ 2b - 7a = 0 \Rightarrow b = 21/4 \end{cases}$$

$$\therefore a_n = \alpha 3^n + \beta 2^n - n \cdot 2^{n+1} + \frac{3}{2}n + \frac{21}{4}$$

[40]

$$\begin{cases} a_n = 3a_{n-1} + 2b_{n-1} \\ b_n = a_{n-1} + 2b_{n-1} \end{cases} \quad \begin{array}{l} a_1 = 3 + 2 \cdot 2 = 7 \\ b_1 = 1 + 2 \cdot 2 = 5 \end{array}$$

$$\underbrace{\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}}_A \begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix}, \text{ d\"ar } \bar{x}_0 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} 3-\lambda & 2 \\ 1 & 2-\lambda \end{vmatrix} = (3-\lambda)(2-\lambda) - 2 =$$
$$= 6 - 5\lambda + \lambda^2 - 2 = \lambda^2 - 5\lambda + 4 = 0 \Rightarrow \left\{ \begin{array}{l} \lambda = \frac{5}{2} + \sqrt{\frac{25}{4} - \frac{16}{4}} = \frac{5}{2} + \frac{3}{2} \\ \lambda = 4, 1 \end{array} \right. \Rightarrow$$

$\lambda = 4$ ger:

$$\begin{pmatrix} -1 & 2 \\ 1 & -2 \end{pmatrix} \xrightarrow{0} \bar{x}_1 = t \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$\lambda = 1$ ger:

$$\begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix} \xrightarrow{0} \bar{x}_2 = t \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\therefore \bar{x}_n = c_1 \cdot 4^n \begin{pmatrix} -1 \\ 1 \end{pmatrix} + c_2 \cdot 1^n \begin{pmatrix} 2 \\ 1 \end{pmatrix} \Rightarrow \text{BU ger:}$$

$$\bar{x}_0 = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \Rightarrow c_1 = c_2 = 1$$

svar:

$$\begin{cases} a_n = 2 \cdot 4^n - 1 \\ b_n = 4^n + 1 \end{cases}$$

