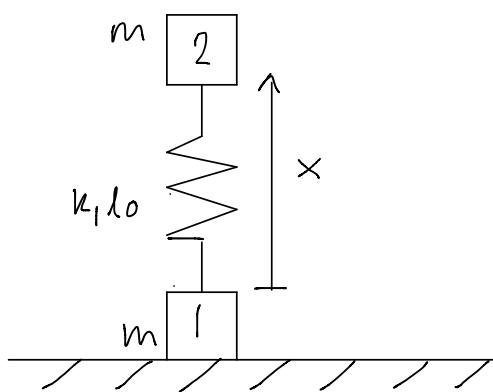


Föreläsning 8

TMME12 – Mekanik I

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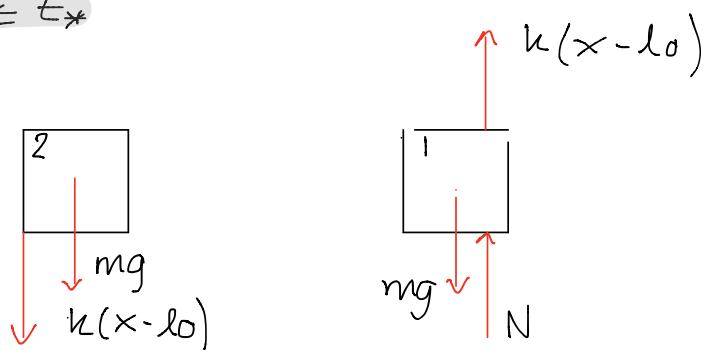
$$G: k = \frac{4mg}{l_0}$$

$$x(0) = \frac{l_0}{4}, \quad \dot{x}(0) = 0$$

S: t_* då kontakt förloras

• Förslagning

$$t \leq t_*$$



Newton II

$$2 \uparrow: -mg - k(x - l_0) = m \ddot{x} \quad (1)$$

$$1 \uparrow: k(x - l_0) - mg - N = m \cdot 0 = 0 \quad (2)$$

ty x def \uparrow

jämvikt.

$$(1) \Rightarrow \ddot{x} + \frac{k}{m}x = -g + \frac{kl_0}{m} \quad (3)$$

$$x = x_h + x_p$$

$$x_p = C$$

Insättning i ekvation (3) \Rightarrow

$$\Rightarrow \frac{k}{m}C = -g + \frac{kl_0}{m} \Leftrightarrow C = -\frac{mg}{k} + l_0 = \frac{3l_0}{4}$$

$$x_h = A \cos \omega_n t + B \sin \omega_n t$$

$$\dot{x}_h = -A \omega_n \sin \omega_n t + B \omega_n \cos \omega_n t$$

Begynnelsevillkor:

$$x(0) = \frac{l_0}{4} \Rightarrow A + \frac{3l_0}{4} = \frac{l_0}{4}$$

$$\Leftrightarrow A = -\frac{l_0}{2}$$

$$\dot{x}(0) = 0 \Rightarrow B \omega_n = 0 \Rightarrow B = 0$$

$$\therefore x = -\frac{l_0}{2} \cos \omega_n t + \frac{3l_0}{4}$$

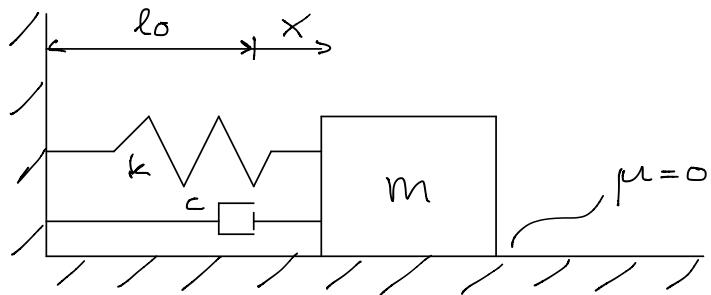
Hela x står uppfylla villkoren.

$$(2) \Rightarrow N = mg - k(x - l_0) = \dots = 2mg(1 + \cos \omega_n t)$$

$$N=0 \Rightarrow \cos \omega_n t_* = -1 \Rightarrow \omega_n t_* = \pi$$

$$\Leftrightarrow t_* = \pi \sqrt{\frac{m}{k}} = \frac{\pi}{2} \sqrt{\frac{l_0}{g}}$$

(84)

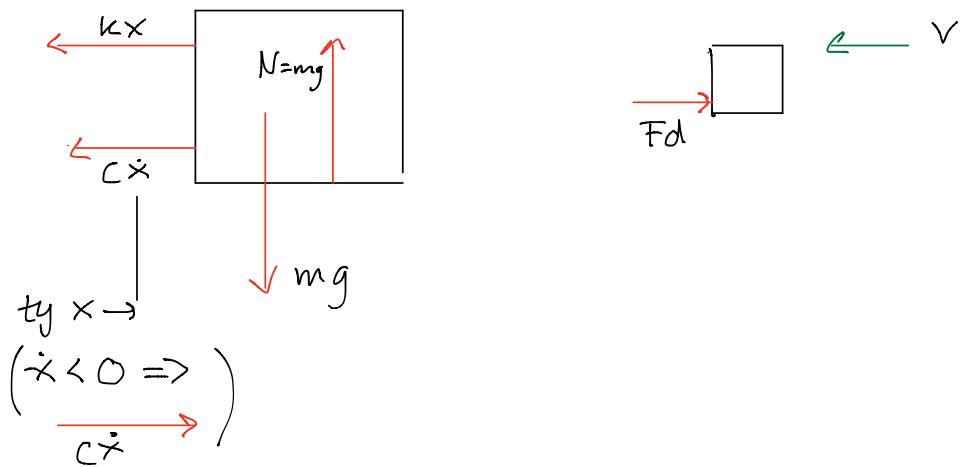


$$q: \ddot{x}(0) = \frac{l_0}{2}, \dot{x}(0) = 0, \delta = 1$$

s: l då vänder.

Frilagsning

Efter $t=0$



Newton VI

$$\rightarrow: -kx - c\dot{x} = m\ddot{x} \Leftrightarrow \ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$$

$$\delta = \frac{2\pi J}{\sqrt{1-J^2}} \Rightarrow J = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}} < 1$$

∴ underdämpat

$$x = x_h = (A \cos \omega_d t + B \sin \omega_d t) e^{-j\omega_n t},$$

$$\omega_n = \sqrt{1 - J^2} \omega_d$$

$$\dot{x} = (-A \omega_d \sin \omega_d t + B \omega_d \cos \omega_d t) e^{-j\omega_n t} - J \omega_n (A \cos \omega_d t + B \sin \omega_d t) e^{-j\omega_n t}$$

BV:

$$x(0) = \frac{\ell_0}{2} \Rightarrow A = \frac{\ell_0}{2}$$

$$\dot{x}(0) = 0 \Rightarrow B \omega_d - J \omega_n A = 0$$

$$\Leftrightarrow B = \frac{J \omega_n \ell_0}{2 \omega_d}$$

$$\therefore x = \left(\frac{\ell_0}{2} \cdot \cos \omega_d t + \frac{J \omega_n \ell_0}{2 \omega_d} \cdot \sin \omega_d t \right) e^{-j\omega_n t}$$

1 vändlåget är $\dot{x} = 0$

$$\Rightarrow \dot{x} = \left(-\frac{\ell_0}{2} \omega_d \sin \omega_d t + \frac{J \omega_n \ell_0}{2} \cos \omega_d t \right) e^{-j\omega_n t}$$

$$- J \omega_n^2 \left(\frac{\ell_0}{2} \cos \omega_d t + \frac{J \omega_n \ell_0}{2} \sin \omega_d t \right) \cdot e^{-j\omega_n t} = 0$$

$$\Leftrightarrow \sin \omega_d t = 0 \Rightarrow \omega_d t = n\pi$$

$n=1$: första vändlåget.

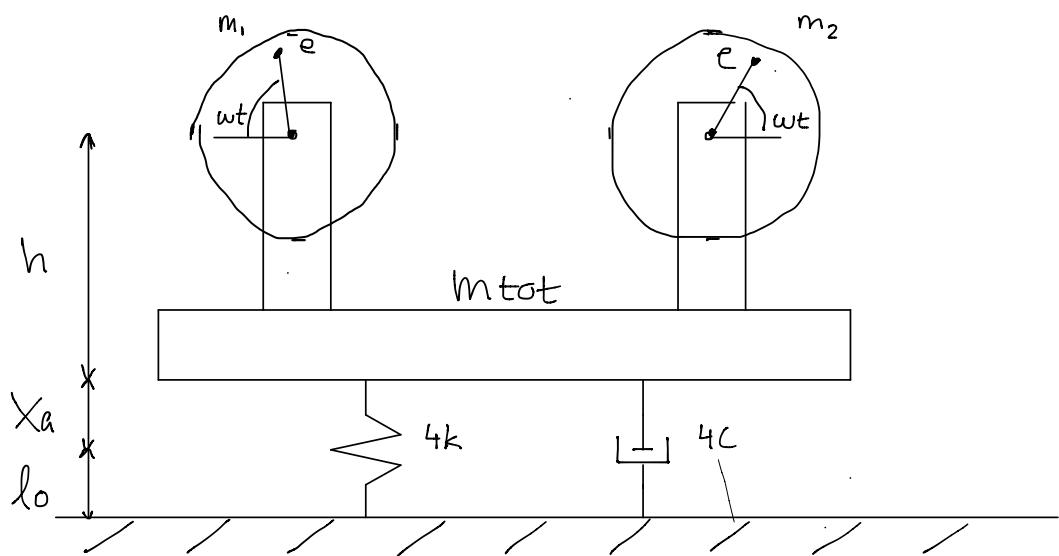
$$t = \frac{n\pi}{\omega_d} \quad (\text{Naturligtns } t = \frac{T_d}{2}, \text{ 1 period}).$$

$$(1) \Rightarrow x \left(\frac{\pi}{\omega_d} \right) = -\frac{l_0}{2} e^{\frac{-\sqrt{J\omega_n\pi}}{\sqrt{1-J^n}} w_n} = -\frac{l_0}{2} e^{-\delta/2} =$$

$$= -\frac{l_0}{2} \cdot \frac{1}{\sqrt{e}}$$

$$\therefore l = l_0 + x = l_0 \left(1 - \frac{1}{2\sqrt{e}} \right)$$

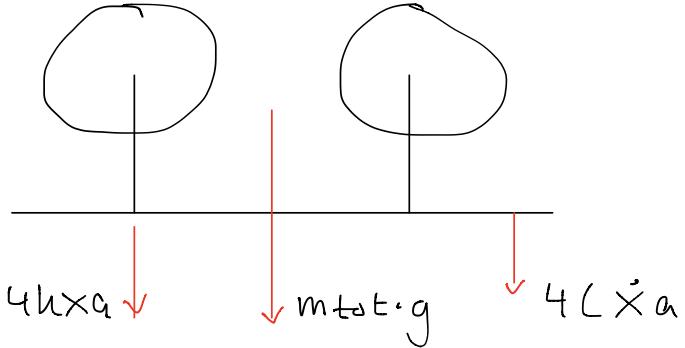
(85)



$$G: k = \frac{m_{tot} \cdot \omega^2}{4}$$

S: x_a i fortvänghet

Frilägg hela anordningen



Newton II

$$F^{ext} = \sum_{i=1}^3 m_i a_i$$

$$\uparrow: -m_{tot}g - 4kx_a - 4L\dot{x}_a = 2m_1(\ddot{x}_a + x_a + h + e \cdot \sin \omega t)$$

$$+ (m_{tot} - 2m_1)(\ddot{x}_a + x_a) = 2m_1(\cancel{\ddot{x}_a} - e\omega^2 \sin \omega t) +$$

$$+ (m_{tot} - 2m_1)\dot{x}_a \Leftrightarrow$$

$$\Leftrightarrow \ddot{x}_a + \frac{4L}{m_{tot}}\dot{x}_a + \frac{4k}{m_{tot}}x_a = \underbrace{\frac{2m_1e\omega^2 \sin \omega t}{m_{tot}}}_{\text{balans kraft.}} - g \quad (1)$$

$$x_a = x_h + x_p$$

Förvarighet ($t \rightarrow \infty$) $\Rightarrow x_a = x_p$

Ansätt $x_p = \underline{x}_1 \sin \omega t + \underline{x}_2 \cos \omega t + \underline{x}_3$

$$\dot{x}_p = \underline{x}_1 \omega \cos \omega t - \underline{x}_2 \omega \sin \omega t$$

$$\ddot{x}_p = -\underline{x}_1 \omega^2 \sin \omega t - \underline{x}_2 \omega^2 \cos \omega t$$

Insättning i (1) \Rightarrow

$$\begin{aligned} & -\cancel{\sum_1 w^2 \sin \omega t} - \cancel{\sum_2 w^2 \cos \omega t} + \\ & + \frac{4L}{m_{\text{tot}}} (\cancel{\sum_1 w \cos \omega t} - \cancel{\sum_2 w \sin \omega t}) + \\ & + \frac{4k}{m_{\text{tot}}} (\cancel{\sum_1 \sin \omega t} + \cancel{\sum_2 \cos \omega t} + \cancel{\sum_3}) = \\ & = \frac{2m_1 \rho \omega^2}{m_{\text{tot}}} \cdot \sin \omega t - g \end{aligned}$$

I dentifiziere:

$$\text{Konstant: } \cancel{\sum_3} = \frac{-m_{\text{tot}} g}{4k}$$

$$\sin \omega t : -\cancel{\sum_1 w^2} - \frac{4L}{m_{\text{tot}}} \cancel{\sum_2 w} + \frac{4k}{m_{\text{tot}}} \cancel{\sum_1} = \frac{2m_1 \rho \omega^2}{m_{\text{tot}}} \quad (2)$$

$$\cos \omega t : -\cancel{\sum_2 w^2} + \frac{4L}{m_{\text{tot}}} \cancel{\sum_1 w} + \frac{4k}{m_{\text{tot}}} \cdot \cancel{\sum_2} = 0 \quad (3)$$

(3) \Rightarrow

$$\cancel{\sum_1} = \frac{w^2 - \frac{4k}{m_{\text{tot}}}}{\frac{4L}{m_{\text{tot}}} \cdot w} \cdot \cancel{\sum_2} = 0$$

(2) \Rightarrow

$$\cancel{\sum_2} = -\frac{m_1 \rho w}{2L}$$

$$\therefore X_a = -\frac{m_1 \rho w}{2L} \cdot \cos \omega t - \frac{m_{\text{tot}} g}{4k} \cdot$$