Lösningsgång

TANA21 – Beräkningsmatematik

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① a) Trunkeringsfel, ~~avrundnings~~.

b) 0,1234 bas 10, 2 dec.

$\Rightarrow \underline{\underline{1,23 \cdot 10^{-1}}}$

c)

$$M\begin{pmatrix}-3\\1\\7\end{pmatrix} = \begin{pmatrix}-3\\0\\0\end{pmatrix} \qquad P = \begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix}$$

$$\underline{\underline{M = \begin{pmatrix}1&0&0\\1/3&1&0\\1/7&0&1\end{pmatrix}}}$$

d) Vid en interpolation söks en funktion som går genom givna punkter.
Vid approximation anpassas en funktion av en viss form så den sluter an till givna punkter så bra som möjligt.

e) Se formelsamling.

②

arean: $A = \pi a b$ $\quad a = 3,4 \pm 0,1$

och $b = 1,1 \pm 0,2$ $\quad \pi = 3,14 \pm 0,005$

$\bar{A} = 11,7436$

$$|\Delta A| = \left|\frac{\partial}{\partial \pi}\Delta\pi\right| + \left|\frac{\partial}{\partial a}\Delta a\right| + \left|\frac{\partial}{\partial b}\Delta b\right)$$

$= |ab \cdot 0{,}605| + |\pi b \cdot 0{,}1| + |\pi a \cdot 0{,}2|$

$\approx 2{,}4993$

$|RB| \leq 0{,}3$

Så $A = 12 \pm 2{,}8$

3 a)

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 2 & 3 & -2 & 0 \\ 3 & 1 & 0 & -6 \\ -3 & -3 & 3 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 3 & 1 & 0 & -6 \\ 2 & 3 & -2 & 0 \\ -3 & -3 & 3 & 0 \end{array}\right)$$

$\left(-\frac{2}{3}\right) \cdot (1) + (2)$

$1 \cdot (1) + (3)$

$$\sim \begin{pmatrix} 3 & 1 & 0 \\ 0 & 2{,}33 & -2 \\ 0 & -2 & 3 \end{pmatrix} \frac{6}{7} \sim \begin{pmatrix} 3 & 1 & 0 \\ 2/3 & 2{,}33 & -2 \\ -1 & -\frac{6}{7} & 1{,}28 \end{pmatrix}$$

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad U = \begin{pmatrix} 3 & 1 & 0 \\ & 2{,}33 & -2 \\ & & 1{,}28 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & & \\ 2/3 & 1 & \\ -1 & -6/7 & 1 \end{pmatrix}$$

$L\underbrace{Uy}_{x} = Pb \Rightarrow Uy = x$

$Lx = Pb$

$$\left(\begin{array}{ccc|c} 1 & & & -6 \\ 2/3 & 1 & & 6 \\ -1 & -6/7 & 1 & 0 \end{array}\right)$$

$x_1 = -6$

$x_2 = 0 + \frac{2}{3} \cdot (6) = 4$

$x_3 = 0 + \frac{6}{7} \cdot 4 - 6 = 2{,}57\ldots$

$$\left(\begin{array}{ccc|c} 3 & 1 & 0 & -6 \\ & 2{,}33 & -2 & 4 \\ & & 1{,}28 & -2{,}57 \end{array}\right) \Rightarrow$$

$y_1 = -2$

$y_2 = \frac{4+2 \cdot (+2)}{2{,}33} = 0$

$y_3 = -2$

Sei $x = \begin{pmatrix} -2 \\ 0 \\ -2 \end{pmatrix}$

c) $\|\Delta b\|_\infty \leq \frac{1}{2}$ $\quad \|A^{-1}\|_\infty = \frac{16}{9}$

$\|A\|_\infty = 9$

$$\frac{\|\Delta x\|_\infty}{\|x\|_\infty} \leq \|A^{-1}\|_\infty \cdot \|A\|_\infty \cdot \frac{\|\Delta b\|_\infty}{\|b\|_\infty} \leq \frac{4}{3}$$

4

x	3,0	4,0	5,0
$f(x)$	-0,35	0,50	-0,15

Newton!

$p(x) = c_1 + c_2(x-4)$

$p(4) = c_1 \qquad = 0{,}5$

$p(5) = c_1 + c_2 = -0{,}15$

$c_1 = 0{,}5$

$c_2 = -0{,}65$

$\Rightarrow p(x) = 0{,}5 - 0{,}65(x-4) = \underline{\underline{0{,}305}}$

$f(4,3)$

b) $p(x) = c$

$p(3) = c_1$
$p(4) = c_1$
$p(5) = c_1$

$\Rightarrow A = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad A^T = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \quad b = \begin{pmatrix} -0{,}35 \\ 0{,}5 \\ -0{,}15 \end{pmatrix}$

$3c_1 = 0 \Rightarrow c_1 = 0$

sai $f(4,3) \approx p(4,3) = 0$

5 $y(1) \quad h = \frac{1}{2}$

$y'' = y' - y + x \quad y(0) = -1, \; y'(0) = 0$

$\begin{cases} u = y \\ v = y' \end{cases} \Rightarrow \begin{cases} u' = v \\ v' = v - u + x \end{cases} \quad y = \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$y_{i+1} = y_i + h\, f(x_i, y_i) \Rightarrow$

$y_1 = \begin{pmatrix} -1 \\ 0 \end{pmatrix} + \frac{1}{2} f\left(0, \begin{pmatrix} -1 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} -1 \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1/2 \end{pmatrix}$

$$y_2 = \begin{pmatrix} -1 \\ 1/2 \end{pmatrix} + \frac{1}{2} f\left(\frac{1}{2}, \begin{pmatrix} -1 \\ 1/2 \end{pmatrix}\right) = \begin{pmatrix} -1 \\ 1/2 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1/2 \\ 2 \end{pmatrix} = \begin{pmatrix} -3/4 \\ 3/2 \end{pmatrix}$$

Så, $y(1) = \underline{-0{,}75}$

$t(n) \approx Cn^p$

6

a)

n	512	1024	2048	4096	8192
t	0,0875	0,299	1,79	12,3	110

$t(2n)/t(n) \approx 2^p \Rightarrow$

$t(1024)/t(512) = 3{,}417\ldots$

$t(8192)/t(4096) = 8{,}9$

$\underline{p = 3}$ ja som forväntad.

b) Runges fenomen!

c) Ansätt $F(h) \approx ch^p$

$$\frac{f(4h) - f(2h)}{f(2h) - f(h)} \approx 2^p$$

$\Rightarrow 2^p = 16 \Leftrightarrow \underline{p = 4}$

Forväntad?