Föreläsning 7

TMME04 – Mekanik II

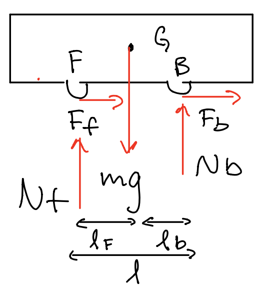
Skriven av Oliver Wettergren
oliwe188@student.liu.se
https://www.instagram.com/olwettergren/

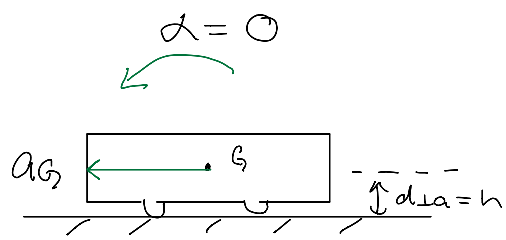
30

Sökt: Max retardation för motorcykel

$|F_{fr}| \leq \mu N$

Frilägg





Euler I,

$\bar{F} = m\bar{a}_G$

$\leftarrow$: $-F_f - f_b = m a_G$ (1)

$\uparrow$: $N_f + N_b - mg = 0$ (2)

Euler II, (skalärt)

$M_B = I_G \alpha + m a_G d_{\perp a}$

B: $N_f \cdot l - mg \cdot l_b = I_G \alpha - m a_G h = 0 - m a_G h$ (3)

(1) och (3) $\Rightarrow$

$N_f = mg \dfrac{l_b}{l} + \dfrac{F_f + F_b}{l} \cdot h$ (4

(2) $\Rightarrow$

$$N_b = \frac{mg}{l}(l - l_b) - \frac{F_f + F_b}{l} \cdot h \quad (5)$$

Retardation, r.

$$r = -a_G \overset{(1)}{=} \frac{F_f + F_b}{m} \quad (6)$$

$F_f \leq \mu N_f$, $F_b \leq \mu N_b$,

så r_{max} fås då

$F_f = \mu N_f$, $F_b = \mu N_b$ $(N_b > 0)$

$$r_{max} \overset{(4),(5)}{=} \frac{\mu}{m}\left(mg\frac{l_b}{l} + mg\frac{(l - l_b)}{l}\right) = \mu g$$

Måste ha $N_b > 0$

$$(5) \Rightarrow \frac{mg(l - l_b)}{l} - \underbrace{\frac{F_f + F_b}{l} \cdot h}_{\overset{(6)}{=} \frac{mrh}{l}} > 0 \Leftrightarrow$$

$$r < \frac{l - l_b}{h} \cdot g = \frac{l_f}{h} \cdot g$$

$\because$ $r_{max} = \min\left(\mu g, \frac{l_f}{h} \cdot g\right)$

Om $\mu > \frac{l_f}{h}$ begränsas alltså r_{max} av tippning.

b) Sökt: $\frac{F_f}{F_{tot}}$,

$F_{tot} := F_f + F_b$

då r_{max}.

Givet: $N_b > 0$

Vid r_{max} gäller

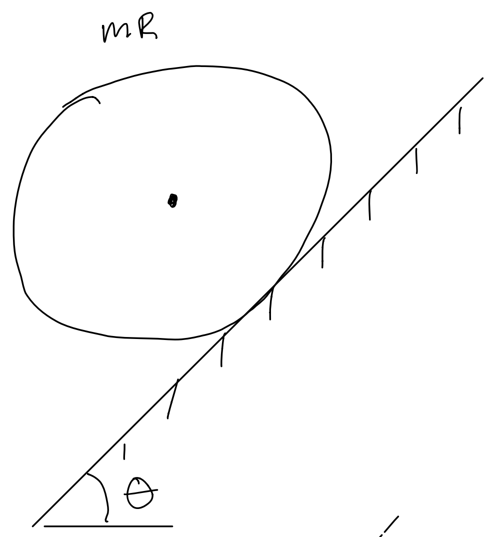
$$\frac{F_f}{F_{tot}} \overset{(4)}{=} \frac{\mu}{F_{tot}}\left(mg\frac{l_b}{l} + \frac{F_{tot}}{l}\cdot h\right) = \quad \overset{(6)}{F_{tot} = m r_{max}} = m\mu g$$

$$= \frac{l_b + \mu h}{l}$$

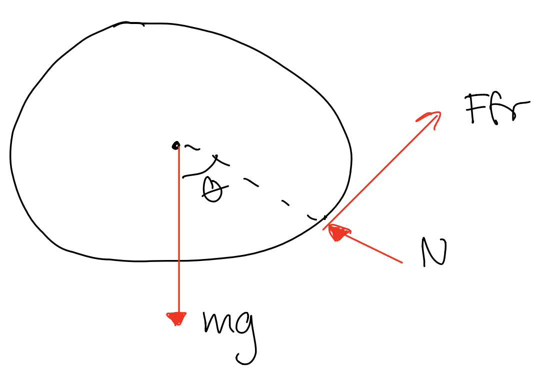
31

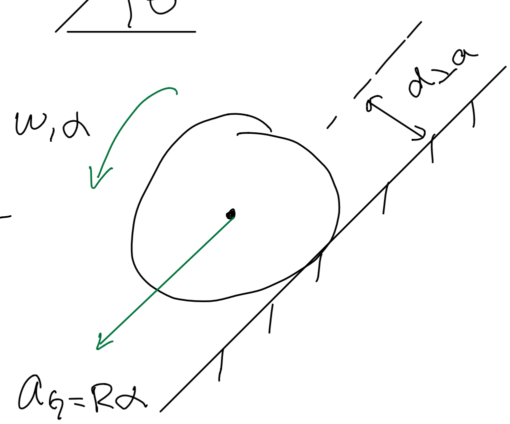
Givet: Ingen glidning

Sökt: a_G



Frilägg:





Ingen glidning, plant, fixt underlag

Euler II (skalärt)

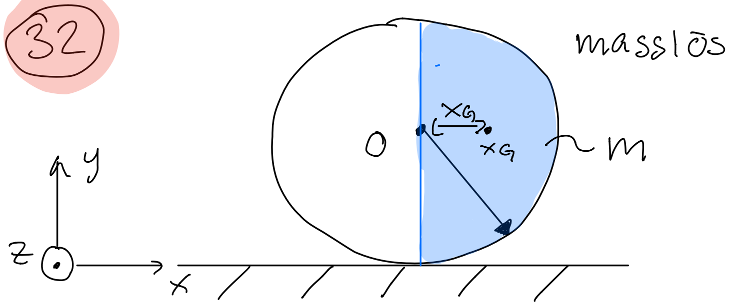
$$M_A = I_G \alpha + m a_G d_{\perp a}$$

$\circlearrowleft A$: $mgR\sin\theta = \underbrace{I_G}_{\frac{mR^2}{2}\text{ (tabell)}} \alpha + mR\alpha \cdot R \iff$

$$\iff \alpha = \frac{2\sin\theta}{3} \cdot \frac{g}{R}$$

$$\therefore\ a_G = \frac{2\sin\theta}{3} \cdot g,\ \swarrow$$

32



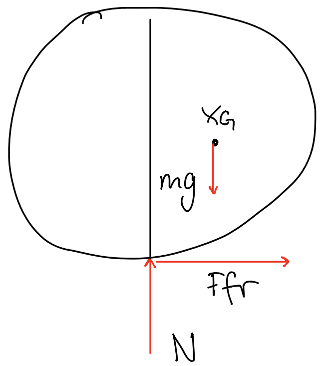
Givet: Släpps från vila vid $t=0$, Ingen glidning,

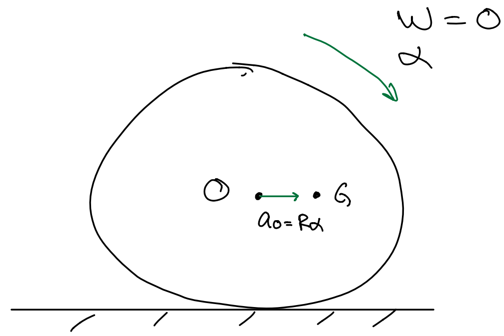
$$x_G = \frac{4R}{3\pi}$$

Sökt: α vid $t=0$, μ_s min så ej börjar glida

$$|F_{fr}| \leq \mu_s N$$

Frilägg:





Euler I:

$$\bar F = m\bar a_G$$

$$\longrightarrow: \quad F_{fr} = m a_{Gx} \qquad (1)$$

$$\uparrow: \quad N - mg = m a_{Gy} \qquad (2)$$

Euler II: (vektor)

$$\bar M_n = I_G \bar\alpha + \bar r_{nG} \times m \bar a_G, \qquad \cancel{M}$$

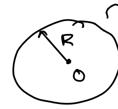
$$\bar M_n: \quad -mg\, x_G \hat z = -I_G \alpha \hat z + (x_G \hat x + R\hat y) \times m\bar a_G \qquad (3)$$

Kinematik

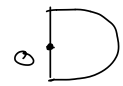
$$\bar a_G = \underbrace{\bar a_o}_{R\alpha\hat x} + \underbrace{\bar\alpha}_{-\alpha\hat z} \times \underbrace{\bar r_{oG}}_{x_G\hat x} - \underbrace{\omega^2}_{=0} \bar r_{oG} = R\alpha\hat x - x_G\alpha\hat y$$

Insättning i (3) => .... =>

~ cirkelskiva



$M = 2m$



m

$$\alpha = \frac{m g x_G}{\underbrace{I_G + m x_G^2 + mR^2}_{I_O,\ \text{Huygens}}} = \left[I_O = \frac{1}{2} M R^2 \qquad I_O = \frac{1}{2}\left(\frac{1}{2} M R^2\right) = \frac{1}{2} m R^2 \right]$$

$$= \frac{m g x_G}{\frac{3}{2} m R^2} = \frac{8g}{9\pi R},$$

(1) =>

$$F_{fr} = m a_{Gx} = m R \alpha = \frac{8}{9\pi} m g$$

(2) =>

$$N = mg + m a_{Gy} = mg - m x_G \alpha = mg\left(1 - \frac{32}{27\pi^2}\right)$$

Börjar ej glida om

$$|F_{fr}| \leq \mu_s N \Rightarrow \mu_s \geq \frac{|F_{fr}|}{N} = 0{,}32$$

$\therefore \mu_{s,min} = 0{,}32$

I en del uppgifter:

$$\ddot{\theta}\,d\theta = \dot{\theta}\,d\dot{\theta}$$

Vanliga fel

- Ej tar med F_{fr} som rullar utan glidning (3/4 gjorde det).
- Fel i kinematiken