Lektion 3

TANA21 – Beräkningsmatematik

Linjära ekvationssystem

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3,3

$$P_1 \begin{pmatrix} 1 \\ -3 \\ 7 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \\ 7 \end{pmatrix} \Rightarrow P_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P_2 \begin{pmatrix} 1 \\ -3 \\ 7 \end{pmatrix} = \begin{pmatrix} 7 \\ -3 \\ 1 \end{pmatrix} \Rightarrow P_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$P_3 \begin{pmatrix} 1 \\ -3 \\ 7 \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \\ -3 \end{pmatrix} \Rightarrow P_3 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

3,6 B, E

3,7 M_1, M_2, M_3

$$M_1 \begin{pmatrix} 1 \\ -3 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -7 & 0 & 1 \end{pmatrix} \quad M_3 \begin{pmatrix} -3 \\ 1 \\ 7 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 1/3 & 1 & 0 \\ 7/3 & 0 & 1 \end{pmatrix}$$

$$M_2 \begin{pmatrix} 7 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 7/3 & 1 & 0 \\ 0 & \frac{1}{3} & 1 \end{pmatrix}$$

3,9 $x = (1 \;\; -3 \;\; 7)$

$$\|x\|_1 = 1 + 3 + 7 = 11$$

$$\|x\|_2 = (1 + 9 + 49)^{1/2} = \sqrt{59} \approx 7{,}68$$

$$\|x\|_\infty = \underline{\underline{7}}$$

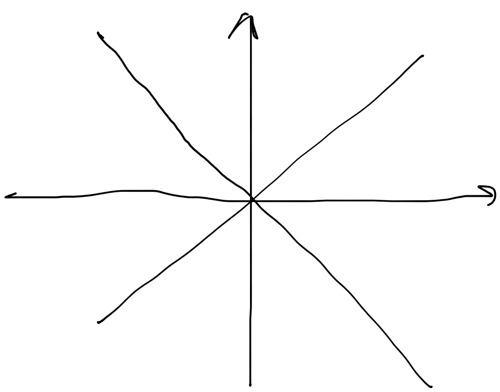
3,12

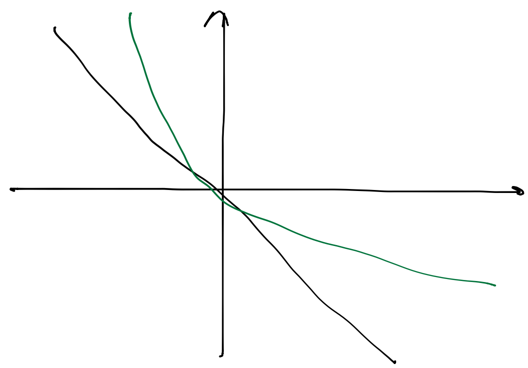
$$A=\begin{pmatrix} 1 & -5 & 3 \\ -4 & 1 & 7 \\ 3 & 2 & -3 \end{pmatrix}$$

$$\|A\|_1 = 3+7+3 = 13 \qquad \|A\|_\infty = 4+1+7 = 12$$

3,15

$$\begin{cases} x_1 + x_2 = 0 \\ x_1 - x_2 = 0 \end{cases} \qquad \begin{cases} x_1 + x_2 = 0 \\ 9x_1 + 10x_2 = 0 \end{cases}$$





Detta är sämre konditionerat ty kan variera mycket vid fel.

3,25

b) $$\begin{cases} -4x_1 = 2 \\ 4x_1 + 2x_2 = 1 \end{cases} \Rightarrow \left(\begin{array}{cc|c} -4 & 0 & 2 \\ 4 & 2 & 1 \end{array}\right) \sim \left(\begin{array}{cc|c} -4 & 0 & 2 \\ -1 & 2 & 3 \end{array}\right)$$

$$U=\begin{pmatrix} -4 & 0 \\ 0 & 2 \end{pmatrix} \quad P=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad L=\begin{pmatrix} 1 & 0 \\ -\frac{1}{2} & 1 \end{pmatrix}$$

$$x_1 = -\frac{1}{2} \quad x_2 = \frac{3}{2}$$

c)

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 0 & -1 & 0 \\ 1 & -2 & 1 & 1 \end{array}\right) \updownarrow \sim \left(\begin{array}{ccc|c} 2 & 0 & -1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & -2 & 1 & 1 \end{array}\right) \left(-\tfrac{1}{2}\right) \downarrow \sim \left(\begin{array}{ccc|c} 2 & 0 & -1 & 0 \\ 1/2 & 1 & 3/2 & 1 \\ 1/2 & -2 & 3/2 & 1 \end{array}\right) \updownarrow$$

$$\sim \left(\begin{array}{ccc|c} 2 & 0 & -1 & 0 \\ 1/2 & -2 & 3/2 & 1 \\ 1/2 & 1 & 3/2 & 1 \end{array}\right) (1/2) \downarrow \sim \left(\begin{array}{ccc|c} 2 & 0 & -1 & 0 \\ 1/2 & -2 & 3/2 & 1 \\ 1/2 & -\frac{1}{2} & 9/4 & 3/2 \end{array}\right)$$

$$x_3 = \frac{3}{2} \cdot \frac{4}{9} = \frac{2}{3} \qquad x_2 = \left(1 - \frac{3}{2} \cdot \frac{2}{3}\right) / -2 = 0 \qquad x_1 = \frac{1}{3}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -1/2 & 1/2 & 1 \end{pmatrix} \quad U = \begin{pmatrix} 2 & 0 & -1 \\ 0 & -2 & 3/2 \\ 0 & 0 & 9/4 \end{pmatrix} \quad P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

d)

$$\left(\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 1 & 1 & -1 & 0 \\ 2 & 5 & 0 & 1 \end{array}\right) \updownarrow \sim \left(\begin{array}{ccc|c} 2 & 5 & 0 & 1 \\ 1 & 1 & -1 & 0 \\ -1 & 1 & 2 & 1 \end{array}\right) (-1/2)\downarrow \ (1/2)\downarrow \sim \left(\begin{array}{ccc|c} 2 & 5 & 0 & 1 \\ 1/2 & -3/2 & -1 & -1/2 \\ -1/2 & 7/2 & 2 & 3/2 \end{array}\right) \updownarrow$$

$$\sim \left(\begin{array}{ccc|c} 2 & 5 & 0 & 1 \\ 1/2 & 7/2 & 2 & 3/2 \\ -1/2 & -5/2 & -1 & -1/2 \end{array}\right) (3/7) \downarrow \sim \left(\begin{array}{ccc|c} 2 & 5 & 0 & 1 \\ 1/2 & 7/2 & 2 & 3/2 \\ -1/2 & -\frac{3}{7} & -\frac{1}{7} & \frac{1}{7} \end{array}\right)$$

$$x_3 = -1 \qquad x_2 = \frac{2}{7}\left(\frac{3}{2} - 2x_3\right) = \frac{2}{7}\left(\frac{3}{2} + \frac{4}{2}\right) = 1$$

$$x_1 = \frac{1 - 5x_2}{2} = -2$$

8

3.26 $Ax = b$

a) $b = (3 \quad 1)^T$

$$A = \begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix} \quad A^{-1} = \begin{pmatrix} 1,5 & 0,5 \\ 2 & 1 \end{pmatrix} \quad \Delta b = \begin{pmatrix} 0,3 \\ 0,1 \end{pmatrix}$$

$$\varkappa_\infty(A) = \|A\|_\infty \cdot \|A^{-1}\|_\infty = (4+3)\cdot(2+1) = \underline{21}$$

$$\varkappa_\infty(b) = \frac{\|\Delta b\|_\infty}{\|b\|_\infty} = \frac{0.3}{3} = 0,1$$

$$\frac{\|\Delta x\|_\infty}{\|x\|_\infty} \le \varkappa(A)\cdot\varkappa(b) = 21 \cdot 0,1 \le \underline{2,1}$$

b) $b = (2,0 \quad 1.0)^T$

$$A = \begin{pmatrix} -4 & 0 \\ 4 & 2 \end{pmatrix} \quad A^{-1} = \begin{pmatrix} -0,25 & 0 \\ 0,5 & 0,5 \end{pmatrix} \quad \Delta b = \begin{pmatrix} 0,05 \\ 0,05 \end{pmatrix}$$

$$\varkappa_\infty(A) = 6 \cdot 1 = 6$$

$$\frac{\|\Delta x\|}{\|x\|} \le \varkappa_\infty(A) \cdot \frac{\|\Delta b\|}{\|b\|} = 6 \cdot \frac{0,05}{2} = 6 \cdot 0,025 = \underline{0,15}$$

d) $b = (1 \quad 3 \quad -3)^T \quad \Delta b = (-0{,}1 \quad 0{,}2 \quad -0{,}1)^T$

$$A = \begin{pmatrix} 3 & 1 & 3 \\ 1 & 3 & -1 \\ 1 & -1 & 1 \end{pmatrix} \quad A^{-1} = \begin{pmatrix} -0{,}25 & 0{,}5 & 1{,}25 \\ 0{,}25 & 0 & -0{,}75 \\ 0{,}5 & -0{,}5 & -1 \end{pmatrix}$$

$$\kappa_\infty(A) = \|A\|_\infty \cdot \|A^{-1}\|_\infty = 7 \cdot 2 = 14$$

$$\frac{\|\Delta x\|}{\|x\|} = \kappa_\infty(A) \cdot \frac{\|\Delta b\|}{\|x\|} = 14 \cdot \frac{0{,}2}{3} \approx 0{,}934\ldots$$

↑

[illegible]