Lektion 6

TAMS24 – Statistisk teori

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Ex:

$$X \sim Exp\left(\frac{1}{\mu}\right):$$

$$f_X(x) = \frac{1}{\mu} e^{-x/\mu}, \quad x > 0$$

observation

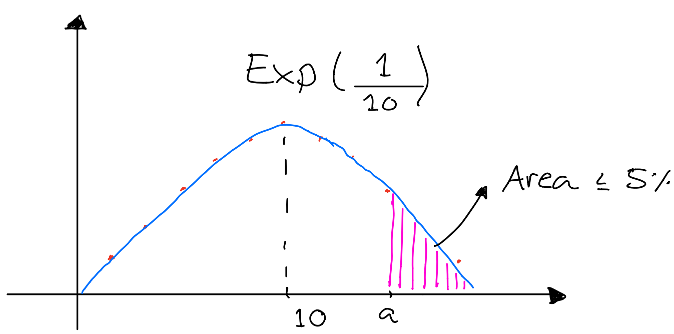
$H_0: \mu = 10$, $H_1: \mu > 10$, $\alpha = 5\%$, $x = 12$

a) Test by c-method ?

b) Test by p-value method ?

c) What is the power if true $\mu = 11$

a)



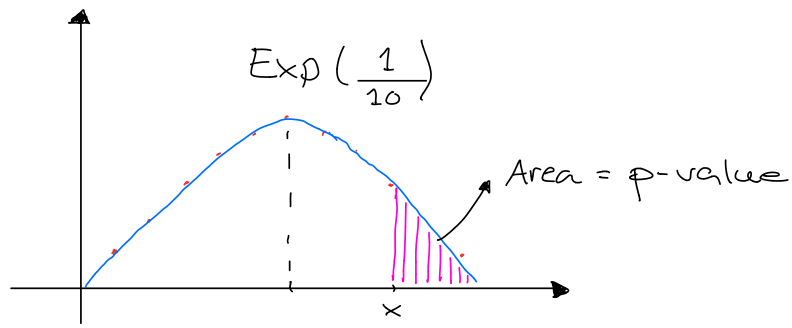
$$C = [a, \infty) = (30, \infty)$$

$$P\left(Exp\left(\frac{1}{10}\right) \geq a\right) \leq 5\%$$

$$\int_a^\infty \frac{1}{10} e^{-x/10} dx \leq 5\% \Rightarrow a = 30$$

$12 \notin C$, don't reject H_0.

b)



p-value $= P\left(Exp\left(\frac{1}{10}\right) \geq x\right) = \int_{12}^{\infty} \frac{1}{10} e^{-x/10} dx \approx$

$\approx 30\% > 5\%.$

Don't reject H_0.

c) $h(11) = P(\text{reject } H_0 \text{ if } H_0 \text{ is false}, \mu = 11) =$

$= P(X \in C \text{ if } \mu = 11) = P\left(Exp\left(\frac{1}{10}\right) \geq 30\right) =$

$= \int_{30}^{\infty} \frac{1}{11} e^{-x/11} dx = 6.5\%.$

13.4 Exponentialdelad s.v.

$X \sim Exp\left(\frac{1}{\theta}\right)$, $E(X) = \theta$

$H_0: \theta = 1000$ $\quad H_1: \theta < 1000$

$x_1 < a$

$P(X < a) = \alpha$

a) a som funktion av α.

$$P(X<a) = P(\text{Exp}(\tfrac{1}{\theta})) = \int_0^a \frac{1}{\theta} e^{-x/\theta} dx =$$

$$= -\left[e^{-x/\theta} \right]_0^a = -(e^{-a/\theta} - 1) = 1 - e^{-a/\theta} = \alpha$$

$$\Rightarrow e^{-a/\theta} = 1 - \alpha \iff -\frac{a}{\theta} = \ln(1-\alpha)$$

$$\iff a = -\theta \ln(1-\alpha) = -1000 \ln(1-\alpha)$$

b) $x_1 = 75$. Resultat signifikant på 5% nivån?

$$a = -1000 \ln(1 - 0{,}05) = 51{,}29\ldots$$

c) $x_1 = 50 < 51{,}3$. Signifikant.

d) $$P(\text{Exp}(\tfrac{1}{\theta}) \leq x) = \int_0^{45} \frac{1}{\theta} e^{-x/\theta} dx = \left[-e^{-x/\theta} \right]_{45}^{\infty} =$$

$$= 1 - e^{-45/1000} = 1 - 0.9559\ldots = 0.044 < 0.05$$

Alltså signifikant.

$\boxed{13.5}$

Okänd sannolikhet p.

Antalet spel X till första vinsten.

$$p_X(k) = p(1-p)^{k-1}, \quad k = 1, 2, 3, \ldots$$

$H_0: p = 0.2, \quad H_1: p < 0.2$

$\alpha = 0.1 = 10\%$

$$P(X > k) = (1-p)^k, \quad k = 0, 1, 2, 3 \ldots$$

$$P(X \geq x \mid H_0) = P(X > x-1 \mid H_0) = (1-p)^{x-1}\Big|_{0.8} = (0.8)^{x-1}$$

Med $x = 1$

$$P(X \geq 11 \mid H_0) = P(X > 10 \mid H_0) = (0.80)^{10} = \underline{\underline{0.1074}}$$

$\therefore$ Förkastar vi ej H_0 på 10% nivån.

13.24

10 täcta behållare. 5 fulla, 5 tomma

Fyra rätt, en fel.

Chansar $\Rightarrow$ hypergeometrisk

$$P(X = k) = \frac{\binom{5}{k}\binom{5}{5-k}}{\binom{10}{5}}, \quad k = 0, 1, 2, \ldots$$

H_0: slagruta värdelös

H_1: slagruta effektiv, H_0 förkastas för stora x.

$x = 4$ ger:

$$P(X \geq 4) = P(X = 4) + P(X = 5) = \frac{\binom{5}{4}\binom{5}{1} + \binom{5}{5}\binom{5}{0}}{\binom{10}{5}} =$$

$$= \frac{25 + 1}{252} = 0.10317$$

$\therefore$ H_0 förkastas ej på 10% nivån.

11~

$H_0: \lambda = 5, \quad H_1: \lambda > 5$

$X \sim Po(\lambda t), \ \lambda = 5, \ \alpha = 0.01, \ \text{Power} = 0.99$

dă

$\lambda = 7.5$

$0.99 = \alpha = P(\text{reject } H_0 \text{ if } H_0 \text{ is false and } \lambda = 7.5) =$

$= P(X > ? \text{ if } \lambda = 7.5) = P\left(\frac{X - \lambda t}{\sqrt{\lambda t}} > \frac{? - \lambda t}{\sqrt{\lambda t}} \text{ if } \lambda = 7.5\right) =$

$= P\left(\frac{X - 7.5t}{\sqrt{7.5}} > \frac{? - 7.5t}{\sqrt{7.5t}}\right) \approx P\left(N(0.1) > \frac{? - 7.5t}{\sqrt{7.5t}}\right),$

$? = 7.5t + \lambda_{0.99}\sqrt{7.5t}$

$5t + \lambda_{0.01}\sqrt{5}\,t = 7.5t + \lambda_{0.99}\sqrt{7.5t}, \quad t = 22.428.$