Föreläsning 2

TMME04 – Mekanik II

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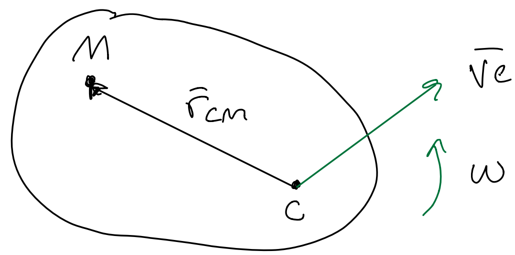
Momentancentrum, plan rorelse

• Givet $\bar{v}_C$ och $\bar{\omega} \neq 0$,

hitta M (momentancentrum)

så att $\bar{v}_M = \bar{0}$:



$$\bar{0} = \bar{v}_M = \bar{v}_C + \bar{\omega} \times \bar{r}_{CM}$$

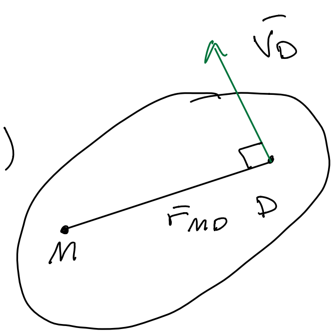
$$\Leftrightarrow \bar{\omega} \times (\bar{v}_C + \bar{\omega} \times \bar{r}_{CM}) = \bar{0}$$

$$\Leftrightarrow \bar{\omega} \times \bar{v}_C - \omega^2 \bar{r}_{CM} = \bar{0}$$

$$\therefore \bar{r}_{CM} = \frac{\bar{\omega} \times \bar{v}_C}{\omega^2}$$

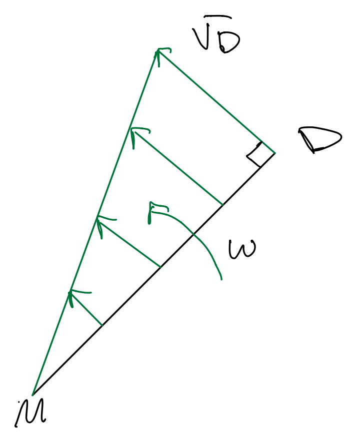
• Givet M, hitta $\bar{v}_D$:

$$\bar{v}_D = \bar{v}_M + \bar{\omega} \times \bar{r}_{MD} = \bar{\omega} \times \bar{r}_{MD} \quad (1)$$



Skalärt:

$$\left| \bar{\omega} \times \bar{r}_{MD} \right| = \omega r_{MD} \cdot \underbrace{\sin\theta}_{=1,\ \theta = 90^\circ}$$



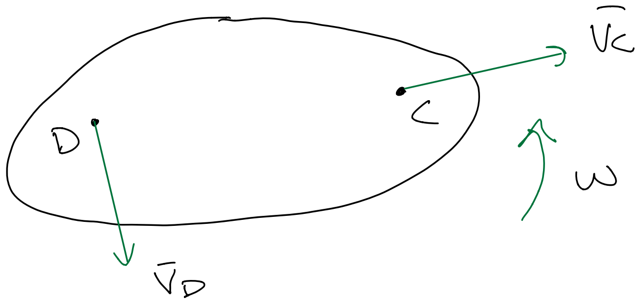
$$\Rightarrow \boxed{v_D = r_{MD}\, \omega}$$

(1) ger att

$$\bar{v}_D \perp \bar{r}_{MD}$$

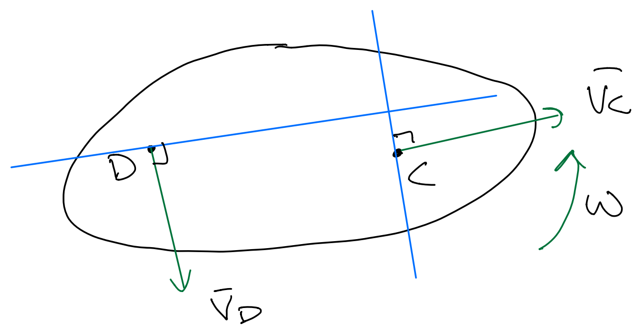
Kroppen roterar momentant runt M.

- Givet C, D, $\bar{V}_C$ och $\bar{V}_D$ hitta ω.

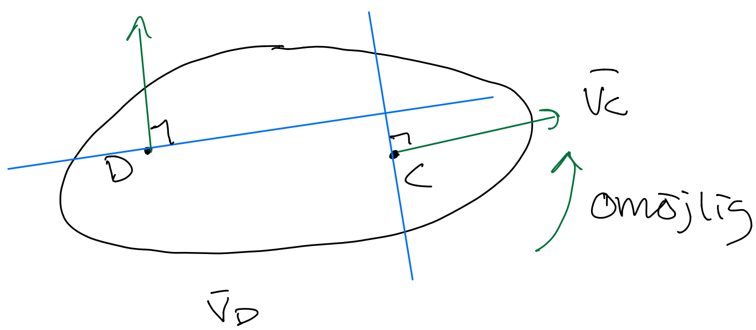


Sök först M grafiskt:

$$\omega = \frac{V_D}{r_{MD}} = \frac{V_C}{r_{MC}}$$

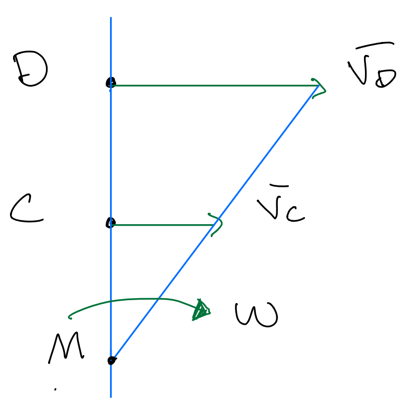


Om det ser ut såhär:



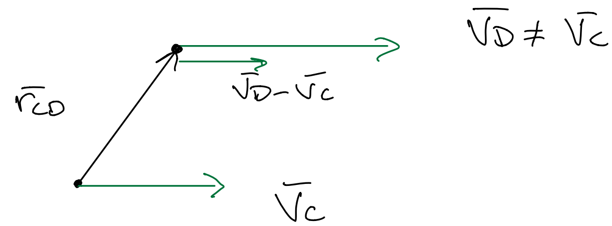
är det en omöjlig stelkroppsrörelse.

Om $\bar{V}_C \parallel \bar{V}_D$:



$(11,25) \Rightarrow$

$\bar{v}_D - \bar{v}_C \perp \bar{r}_{CD}$

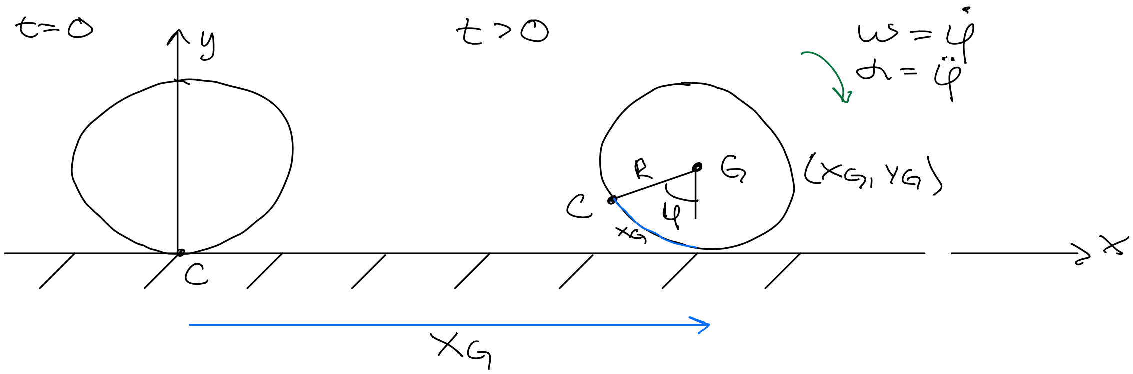


En omöjlig stelkroppsrörelse. Skulle ge ändrat avstånd med tiden.

OK om $\bar{v}_D = \bar{v}_C$: Ren translation, så $\omega = 0$.

Rullande hjul, ingen glidning

- Fixt, plant underlag



$$\begin{cases} x_C = x_G - R\sin\varphi \\ y_C = y_G - R\cos\varphi \end{cases}$$

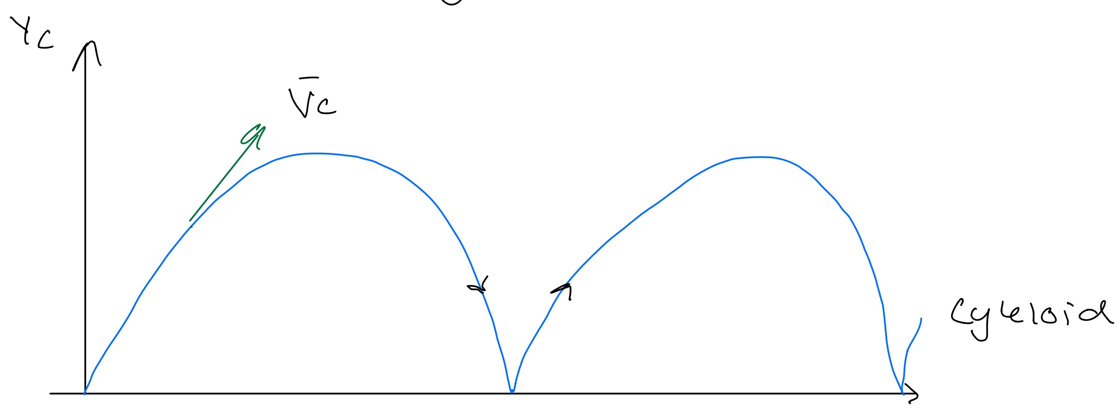
Kinematiska tvång:

$$\begin{cases} y_G = R \\ x_G = R\varphi \end{cases}$$

$\therefore\ \bar{r}_C = x_C \hat{x} + y_C \hat{y} = R(\varphi - \sin\varphi)\hat{x} + R(1-\cos\varphi)$

$$\bar{v}_c = R\omega(1-\cos\varphi)\,\hat{x} + R\omega\sin\varphi\,\hat{y}$$

$$\bar{a}_c = \left[R\alpha(1-\cos\varphi) + R\omega^2\sin\varphi\right]\hat{x} + \left[R\alpha\sin\varphi + R\omega^2\cos\varphi\right]\hat{y}$$



C i kontakt då $\quad \dot{x}_c = 0,\ \dot{y}_c = 0$

$$y_c = 0$$

$$\Rightarrow \quad \varphi = 2\pi n, \quad n = 0, \pm 1, \pm 2, \ldots$$

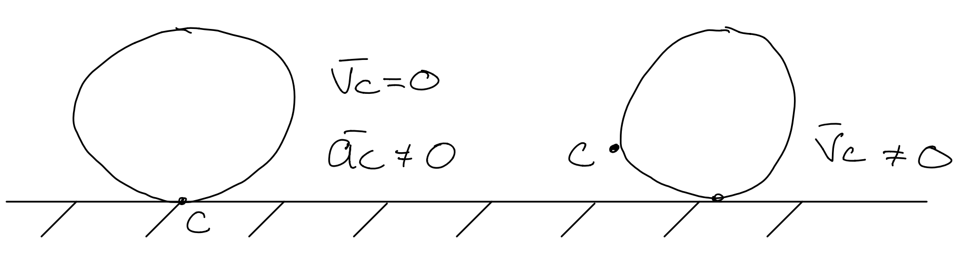
$$\bar{v}_c(\varphi = 2\pi n) = 0$$

∴ Kontaktpunkten är momentancentrum

$$\bar{a}_c(\varphi = 2\pi n) = R\omega^2\hat{y} \quad (\neq 0)$$

t :

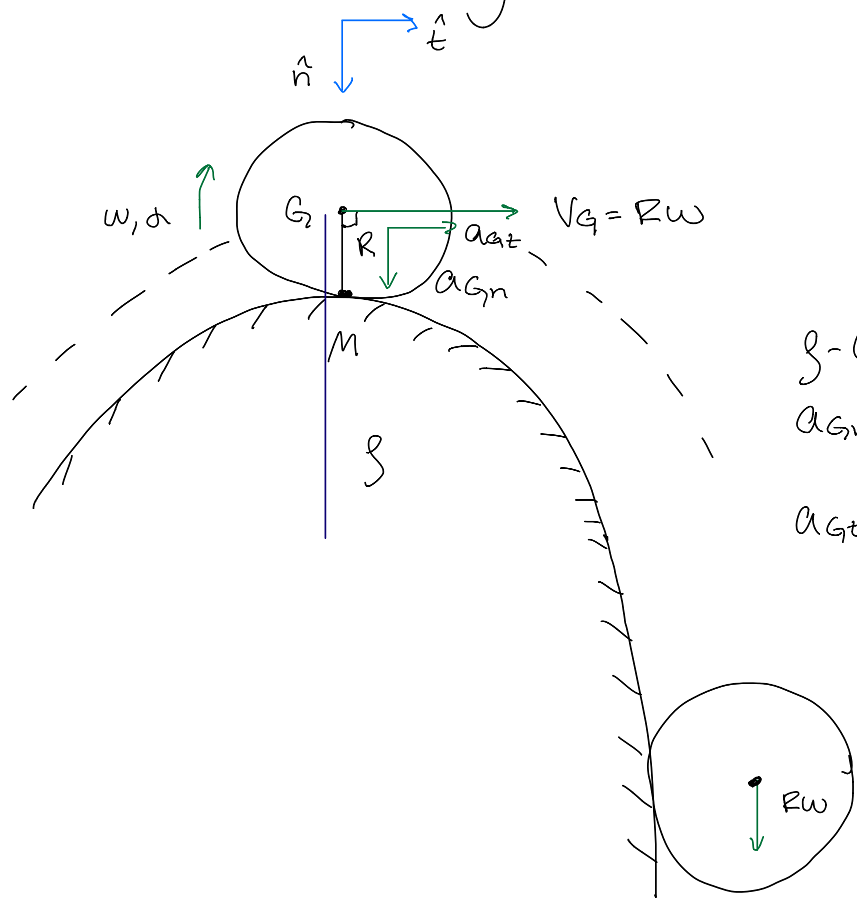
t+dt:



Vid problemlösning utnyttjas

- Kontaktpunkten M
- $V_G = R\omega$
- $a_G = R\alpha$

- Fixt, krökt underlag

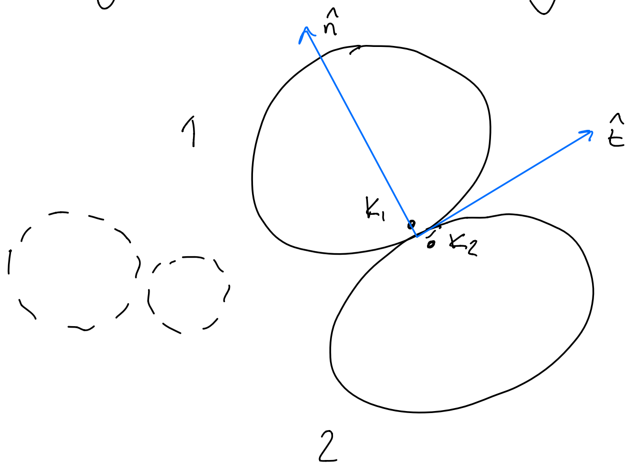


ρ - krökningsradie

$$a_{Gn} = \frac{V_G^2}{\rho}$$

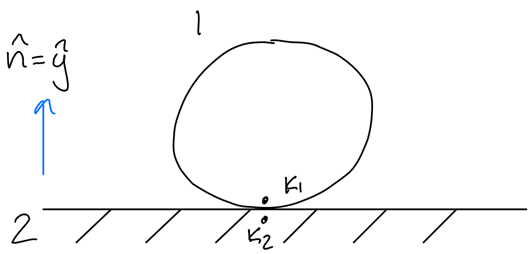
$$a_{Gt} = \dot{V}_G = R\alpha$$

• Rörligt, krökt underlag



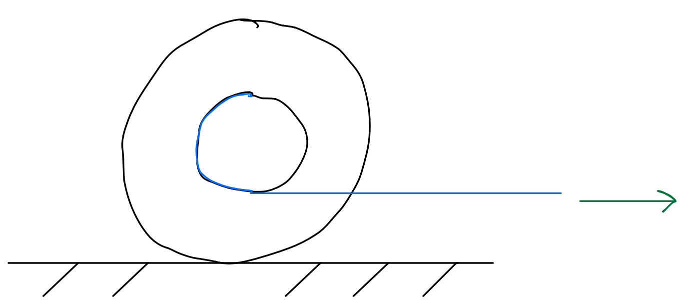
Studera två godtyckligt formade kroppar som rullar mot varandra utan glidning, plan rörelse.

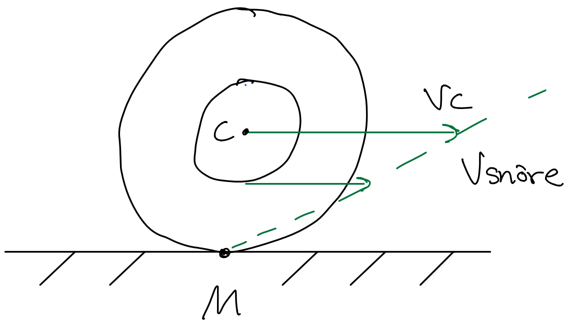
$$\boxed{\begin{array}{l} \bullet\ \bar v_{K_1} = \bar v_{K_2} \\ \bullet\ \bar a_{K_1}\cdot\hat t = \bar a_{K_2}\cdot\hat t \end{array}}$$



$$\bar a_{K_1}\cdot\hat n = R\omega^2\hat y\cdot\hat y =$$
$$= R\omega^2 \neq \underbrace{\bar a_{K_2}}_{=\bar 0}\cdot\hat n$$

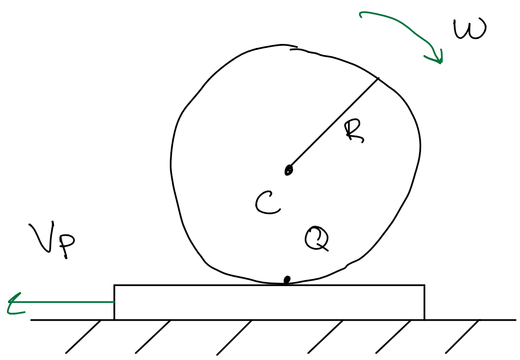
Ex: Åt vilket håll rör sig kabelrullen? Ingen glidning.





$\therefore$ Åt höger!

Ex: Givet $\bar{V}_P$ och $\bar{\omega}$, sök $\bar{V}_C$. Ingen glidning.



$$\bar{V}_Q = \bar{V}_P$$

$$\bar{V}_C = \underbrace{\bar{V}_Q}_{\bar{V}_P} + \bar{\omega} \times \bar{r}_{QC} = R\omega - V_P, \longrightarrow$$