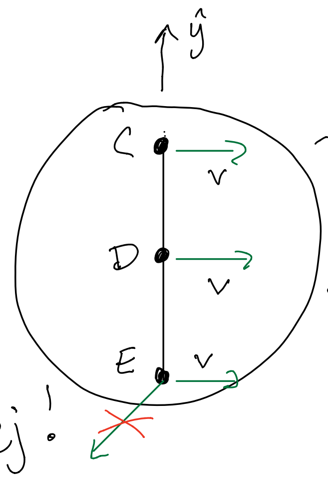
Föreläsning 1

TMME04 – Mekanik II

Skriven av Oliver Wettergren

oliwe188@student.liu.se

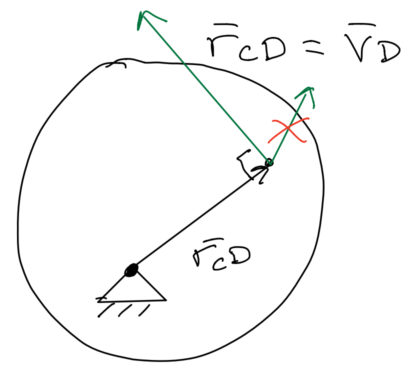
https://www.instagram.com/olwettergren/

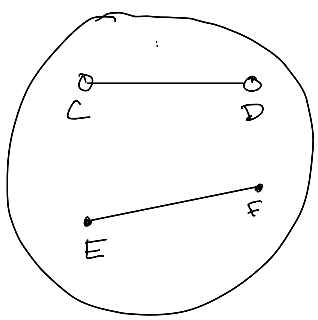


$\Rightarrow \bar{\omega} = \omega \hat{y}$

$\dot{\bar{r}}_{DE} = \omega \times \bar{r}_{DE} = \bar{0}$

går ej!

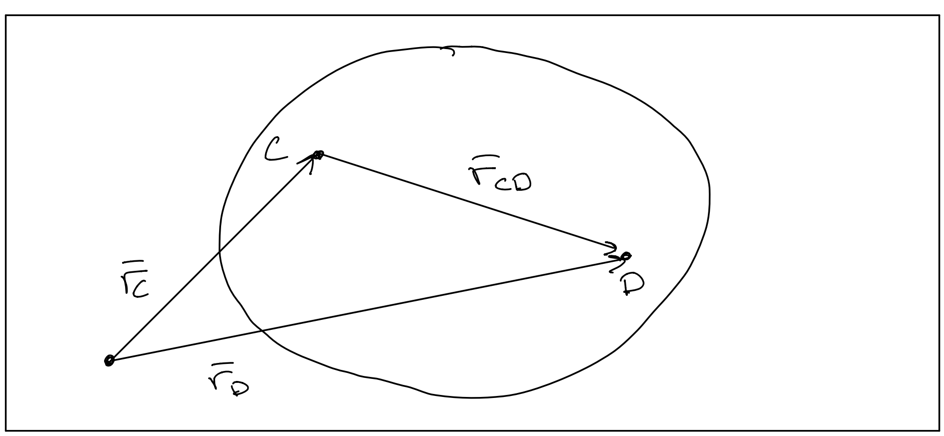




$\dot{\bar{r}}_{CD} = \bar{\omega} \times \bar{r}_{CD}$

$\dot{\bar{r}}_{EF} = \bar{\omega} \times \bar{r}_{EF}$

Hastighets- och accelerationssamband



C, D fixa i kroppen.

$$\dot{\bar{r}} = \bar{\omega} \times \bar{r}_{CD} \qquad (2)$$

(2) ger:

$$\boxed{\bar{v}_D = \bar{v}_C + \bar{\omega} \times \bar{r}_{CD}} \qquad (3)$$

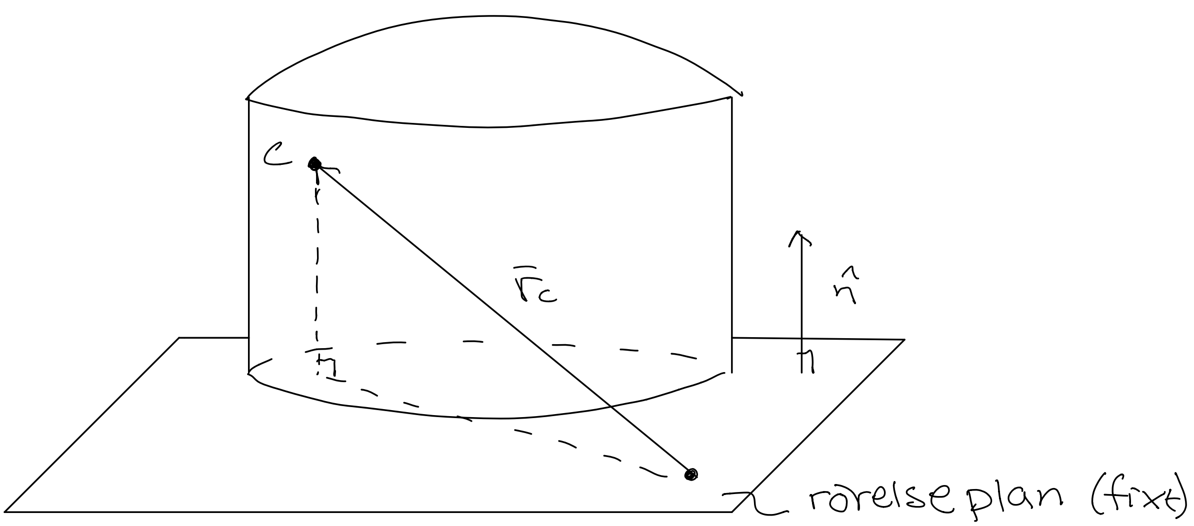
$\bar{\omega}$ kallas vinkelhastighetsvektor för kroppen.

$$\bar{a}_D = \dot{\bar{v}}_D = \dot{\bar{v}}_C + \underbrace{\dot{\bar{\omega}}}_{\bar{\alpha}} \times \bar{r}_{CD} + \bar{\omega} \times \underbrace{\dot{\bar{r}}_{CD}}_{\bar{\omega} \times \bar{r}_{CD} \text{ enligt (2).}}$$

$$\boxed{\bar{a}_D = \bar{a}_C + \bar{\alpha} \times \bar{r}_{CD} + \bar{\omega} \times (\bar{\omega} \times \bar{r}_{CD})} \qquad (4)$$

$\bar{\alpha}$ kallas vinkelaccelerationsvektor för kroppen.

Plan rörelse



Def:

En stel kropp utför plan rörelse om det existerar ett fixt plan (så kallat rörelseplan) med normal $\hat{n}$ sådant att

$$\bar{r}_G(t) \cdot \hat{n} \qquad (5)$$

är konstant i tiden för alla G.

SATS:

Vid plan rörelse gäller

$$\bar{\omega}(t) = \omega(t)\,\hat{n}, \quad \hat{n} \text{ konstant}$$

(riktning konstant, men storlek kan ändras)

Bevis:

(5) ger:

$$0 = \frac{d}{dt}\left(\bar{r}_C \cdot \hat{n}\right) = \bar{v}_C \cdot \hat{n} \Rightarrow$$

$$\Rightarrow \left[\, C \text{ godtycklig, så även } \bar{v}_D \cdot \hat{n} = 0 \,\right] \Rightarrow$$

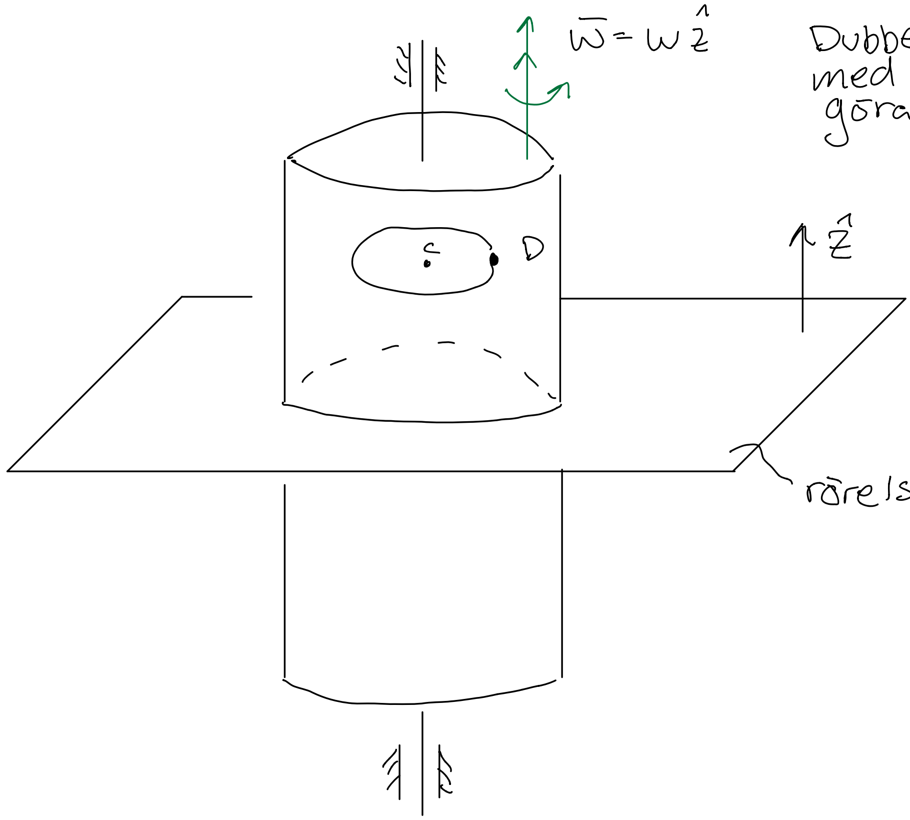
$$\Rightarrow 0 = (\bar{v}_D - \bar{v}_C) \cdot \hat{n} = (\bar{\omega} \times \bar{r}_{CD}) \cdot \hat{n} =$$

$$= \left(\hat{n} \times \bar{\omega}\right) \cdot \bar{r}_{CD} \qquad \forall\, C, D$$

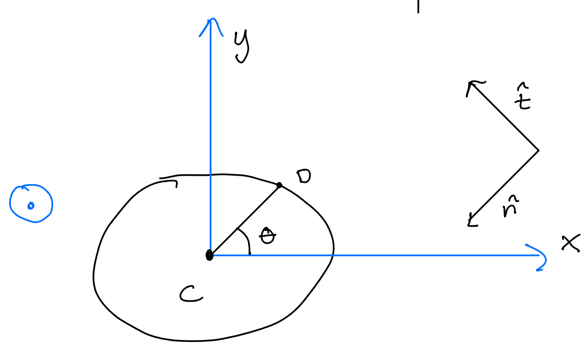
trippelskalärprodukt

$$\Leftrightarrow \hat{n} \times \omega = 0 \Leftrightarrow \bar{\omega} = \omega \hat{n} \qquad \blacksquare$$

Ex: Rotation kring fix axel



Dubbelpil har med rotation att göra.



$$\bar{v}_C = \bar{0}$$

$$\bar{r}_C = -r\hat{n}$$

$$\bar{\omega} = \omega \hat{z}$$

$$\bar{v}_D \overset{(3)}{=} \bar{v}_C + \bar{\omega} \times \bar{r}_{CD} = \omega \hat{z} \times (-r\hat{n}) = \omega r \hat{t}$$

där

$\omega = \dot{\theta}$ vinkelhastighet, jämför med partikelmek.

Om C, D ligger i samma plan $\perp \bar{\omega}$, kan termen $\bar{\omega} \times (\bar{\omega} \times \bar{r}_{CD})$ i (4) förklaras.

$$\bar{a} \times (\bar{b} \times \bar{c}) = \bar{b}(\bar{a} \cdot \bar{c}) - \bar{c}(\bar{a} \cdot \bar{b}).$$

$$\bar{\omega} \times (\bar{\omega} \times \bar{r}_{CD}) = \bar{\omega} \underbrace{(\bar{\omega} \cdot \bar{r}_{CD})}_{\substack{=0 \\ \text{ty } \bar{\omega} \perp \bar{r}_{CD}}} - \bar{r}_{CD} \underbrace{(\bar{\omega} \cdot \bar{\omega})}_{\omega^2} = -\omega^2 \bar{r}_{CD}$$

Insättning i (4) $\Rightarrow$

$$\boxed{\bar{a}_D = \bar{a}_C + \bar{\alpha} \times \bar{r}_{CD} - \omega^2 \bar{r}_{CD}, \quad \bar{\omega} \perp \bar{r}_{CD}} \quad (6)$$

(gäller för alla tal vi ska räkna på).

Ex: Rotation kring fix axel, fortsättning

$\bar{a}_C = \bar{0}$

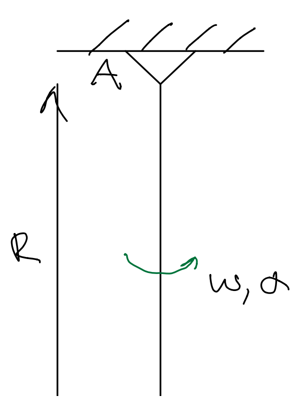
$\bar{\alpha} = \dot{\bar{\omega}} = \alpha \hat{z}$

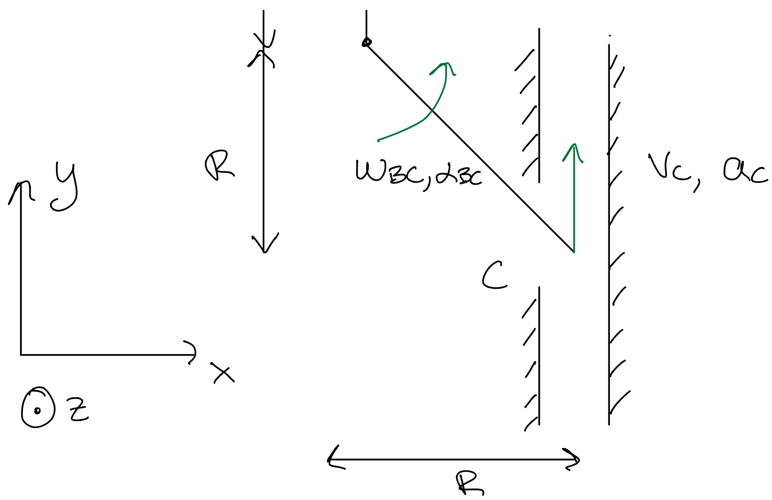
$\bar{a}_D = \bar{\alpha} \times \bar{r}_{CD} - \omega^2 \bar{r}_{CD} = \alpha r \hat{t} + \omega^2 r \hat{n}$

där

$\alpha = \ddot{\theta}$, vinkelacceleration.

Ex:



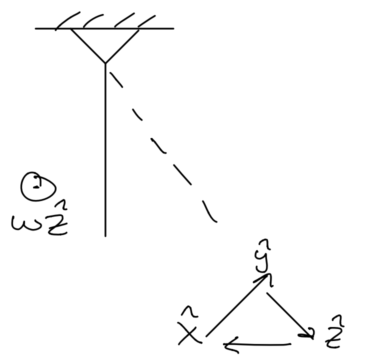


Hastighetsanalys:

- AB:

$$\bar V_B = \underbrace{\bar V_A}_{=0} + \underbrace{\omega_{AB}}_{\omega\hat z} \times \underbrace{\bar r_{AB}}_{-R\hat y}$$

$$\bar V_B = [\text{cirkelrorelse}] = R\omega\hat x \quad (7)$$



$\hat z \times \hat y = -\hat y \times \hat z$

- BC:

$$\bar V_B = \underbrace{\bar V_C}_{V_C\hat y} + \underbrace{\bar\omega_{BC}}_{\omega_{BC}\hat z} \times \underbrace{\bar r_{CB}}_{-R\hat x + R\hat y} = V_C\hat y - R\omega_{BC}\hat y - R\omega_{BC}\hat x \quad (8)$$

Identifiering av (7)=(8):

$\hat x$: $R\omega = -R\omega_{BC} \iff \omega_{BC} = -\omega$

$\hat y$: $0 = V_C - R\omega_{BC} \iff V_C = -R\omega$

$\therefore$ $\omega_{BC} = \omega$, ↻

$\bar V_C = R\omega$, ↓

kan skippa, pga

Accelerationsanalys:

AB:

$$\bar{a}_B = [\text{cirkelrörelse}] = R\alpha\hat{x} + R\omega^2\hat{y} \quad (9)$$

BC:

$$\bar{a}_B = \underbrace{\bar{a}_C}_{a_C\hat{y}} + \underbrace{\bar{\alpha}_{BC}}_{\alpha_{BC}\hat{z}} \times \underbrace{\bar{r}_{CB}}_{-R\hat{x}+Ry} - \underbrace{\omega^2_{BC}}_{(-\omega)^2}\bar{r}_{CB} =$$

kompensera ej för felsatt riktning

$$= a_C\hat{y} - R\alpha_{BC}\hat{y} - R\alpha_{BC}\hat{x} + R\omega^2\hat{x} - R\omega^2\hat{y} \quad (10).$$