Föreläsning 5

TMME04 – Mekanik II

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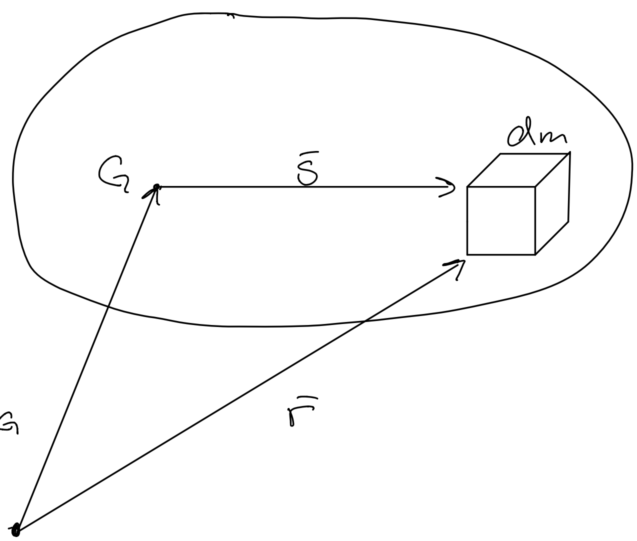
Kinetik



Def: Masscentrum

$$\bar{r}_G = \frac{\int \bar{r}\, dm}{\underbrace{\int dm}_{m}} \qquad (9)$$

$$\int \bar{s}\, dm = \int (\bar{r} - \bar{r}_G)\, dm \overset{(9)}{=} m\bar{r}_G - \bar{r}_G \underbrace{\int dm}_{m} = \bar{0}$$

$$\int \bar{s}\, dm = \bar{0} \quad (10)$$ Viktigt!

Kraftlagen (Euler I)

$$\bar{F} \overset{(7)}{=} \int \ddot{\bar{r}}\, dm \overset{(6)}{=} \left(\int \bar{r}\, dm \right)^{\cdot\cdot} = (m\bar{r}_G)^{\cdot\cdot} \Leftrightarrow \bar{F} = m\bar{a}_G \quad (11)$$

Momentlagen (Euler II)

SATS: förflyttningsregeln för moment

$$\bar{M}_A = \bar{M}_B + \bar{r}_{AB} \times \bar{F}, \quad A, B \text{ godtyckliga} \quad (12)$$

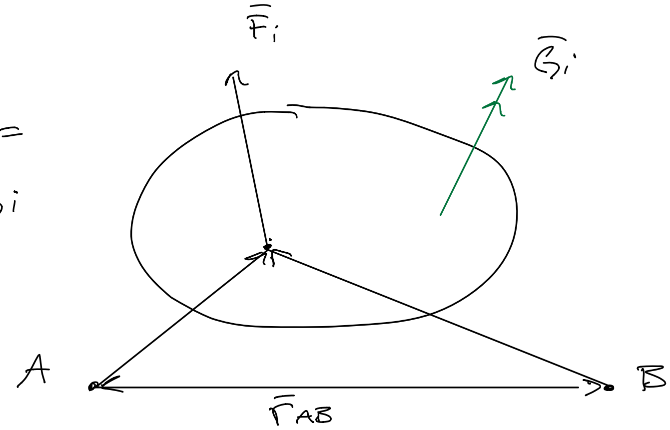
Bevis

$$\bar{M}_A = \sum \bar{\rho}_{Ai} \times F_i + \sum G_i =$$

$$= \sum (\bar{r}_{AB} + \bar{\rho}_{Bi}) \times F_i + \sum G_i$$

$$= \bar{r}_{AB} \times \underbrace{\sum F_i}_{\bar{F}} + \underbrace{\sum \rho_{Bi} \times F_i + \sum \bar{G}_i}_{\bar{M}_B} =$$

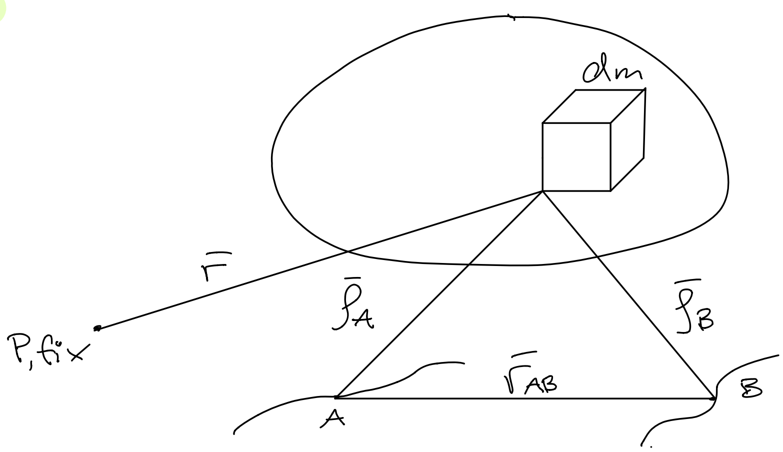
$$= \bar{M}_B + \bar{r}_{AB} \times F \quad \blacksquare$$



SATS: förflyttningsregeln för rörelsemoment

$$\bar{h}_A = \bar{h}_B + \bar{r}_{AB} \times m\bar{v}_G \quad A, B \text{ godtycklig} \quad (13)$$

Bevis



$$\bar h_A = \int \bar\rho_A \times \dot{\bar r}\,dm = \int (\bar r_{AB} + \bar\rho_B)\times \dot{\bar r}\,dm =$$

$$= \bar r_{AB} \times \underbrace{\int \dot{\bar r}\,dm}_{\overset{(6)}{=}\left(\int \bar r\,dm\right)^{G} \underset{q}{=}} + \underbrace{\int \bar\rho_B \times \dot{\bar r}\,dm}_{\bar h_B} = \bar h_B + \bar r_{AB} \times m\bar v_G$$

$$= (m\bar r_G)^{G} = m\bar v_G$$

Omskrivning av momentlagen

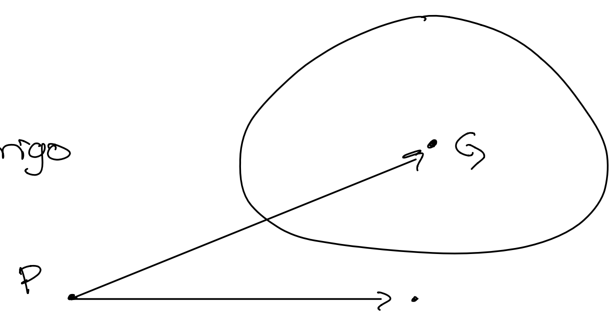
$\bar M_P = \dot{\bar h}_P$ till godtycklig punkt A

Välj B=P i ekvation (12):

$$\bar M_A = \underbrace{\bar M_P}_{\dot{\bar h}_P} + \underbrace{\bar r_{AP}}_{-\bar r_A} \times \underbrace{\bar F}_{m\bar a_G} = \dot{\bar h}_P - \bar r_A \times m\bar a_G \quad (14)$$

Motiveras:

Lägger in origo i P.



fix $\quad \bar r_A = \bar r_{PA} = -\bar r_{AP}$

Välj B=P i ekvation (13):

$$\bar h_A = \bar h_P - \bar r_A \times m\bar v_G \underset{\text{derivera}}{\Rightarrow} \dot{\bar h}_A = \dot{\bar h}_P - \bar v_A \times m\bar v_G - \bar r_A \times m\bar a_G$$

$$\overset{(14)}{=} \bar M_A - \bar v_A \times m\bar v_G$$

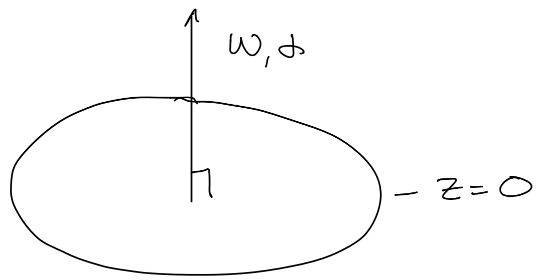
$$\therefore \bar M_A = \dot{\bar h}_A + \bar v_A \times m\bar v_G \quad (15$$

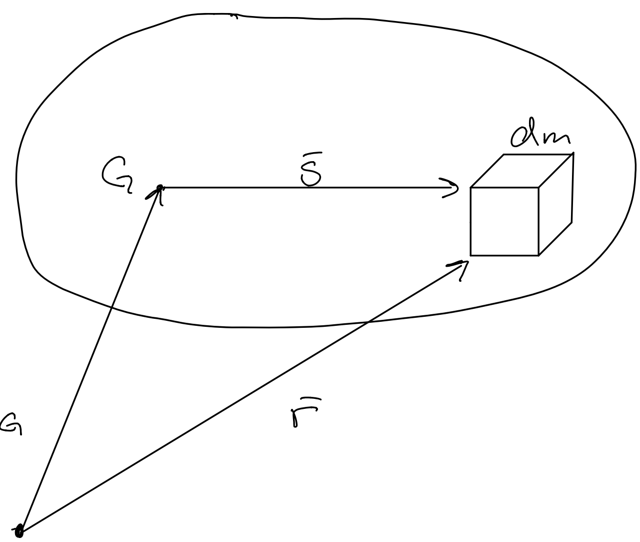
Välj A=G, kan Euler II skrivas som

$$\bar{M}_G = \dot{\bar{h}}_G \quad (16)$$

Bestämning av $\bar{h}_G$ för plana problem

(tunna skivor i fixa planet $z=0$, $\omega = \omega\hat{z}$, $\alpha = \alpha\hat{z}$)





$$\bar{h}_G \overset{(2)}{=} \int \bar{s}\times\dot{\bar{r}}\,dm = \int \bar{s}\times(\dot{\bar{r}}+\dot{\bar{s}})\,dm = \underbrace{\int \bar{s}\,dm}_{\bar{0}\text{ enl. (16)}} \times \bar{v}_G +$$

$$+\int \bar{s}\times\overbrace{(\bar{\omega}\times\bar{s})}^{\bar{s}\text{ kropp fix}}\,dm = \left[\bar{a}\times(\bar{b}\times\bar{c}) = \bar{b}(\bar{a}\cdot\bar{c}) - \bar{c}(\bar{a}\cdot\bar{b})\right]$$

$$= \int \bar{\omega}\underbrace{(\bar{s}\cdot\bar{s})}_{\bar{s}^2}\,dm - \int \bar{s}\underbrace{(\bar{s}\cdot\bar{\omega})}_{0\text{ ty }\bar{s}\perp\bar{\omega}}\,dm = \underbrace{\left(\int s^2\,dm\right)}_{I_G\text{, masströghetsmoment map }G.}\bar{\omega}$$

I_G är tidsberoende $\Rightarrow$

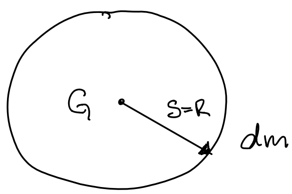
$$\bar{M}_G = I_G \alpha \quad (18)$$

I_G anger kroppens motstånd mot vinkelacceleration.

Ex:

Ring

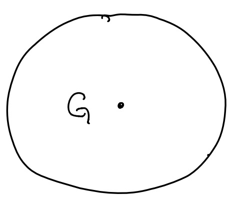
m, R



$$I_G = \int s^2 \, dm = R^2 m \Leftrightarrow I_G = mR^2$$

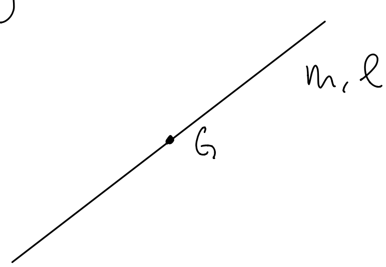
Skiva

m, R



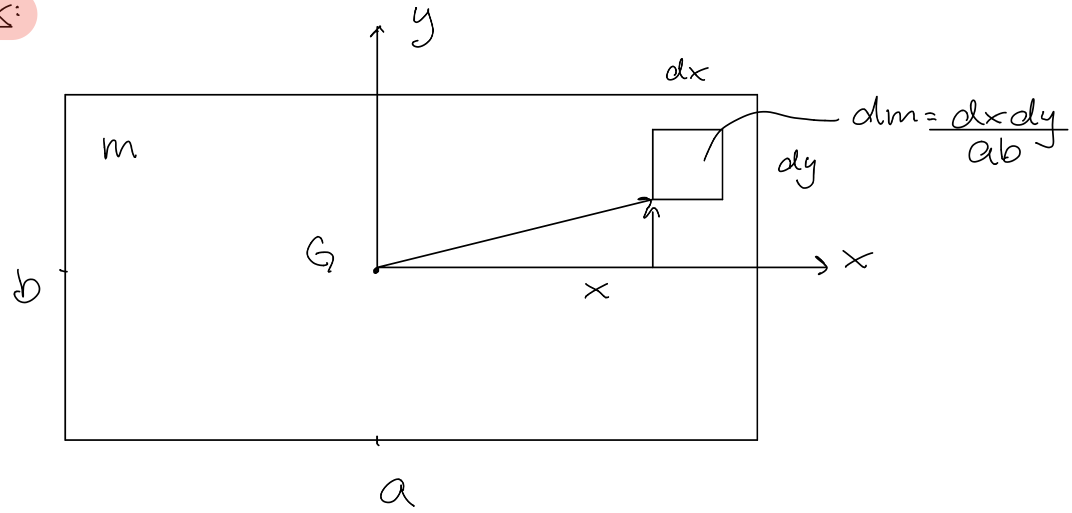
$$I_G = \frac{1}{2} mR^2$$

Stång



$$I_G = \frac{1}{12} m\ell^2$$

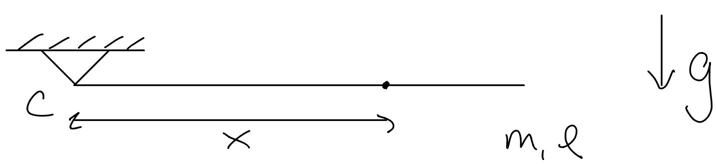
Ex:



$$I_G = \int s^2\,dm = \int_{-b/2}^{b/2}\int_{-a/2}^{a/2} (x^2+y^2)\, m\, \frac{dx\,dy}{ab} = \ldots = \frac{m}{12}(a^2+b^2) \qquad \square$$

$x: -\frac{a}{2} \rightarrow \frac{a}{2}$, $y: -\frac{b}{2} \rightarrow \frac{b}{2}$

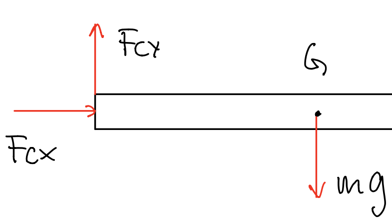
Ex:

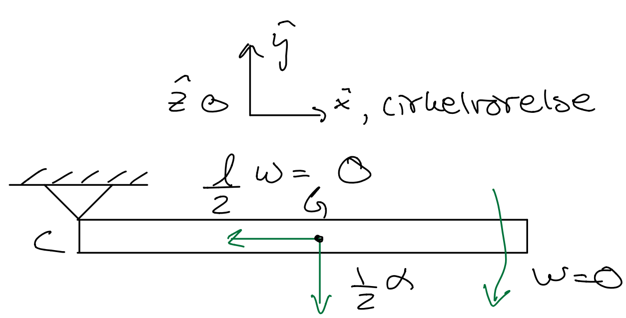


Släpper från vila, vid $t=0$

Sökt: α vid $t=0$

Frilägg vid $t=0$:





Euler I: $\bar{F} = m\bar{a}_G$

$\longrightarrow$: $F_{Cx} = m\bar{a}_{Gx} = 0$

$\uparrow$: $F_{Cy} - mg = m\bar{a}_{Gy} = m\left(-\frac{l}{2}\alpha\right)$ (*)

Euler II: $M_G = I_G\alpha$

$\curvearrowright G$: $F_{Cy}\frac{l}{2} = \frac{ml^2}{12}$, tabell (**)

(*), (**) $\Rightarrow$ $\alpha = \frac{3g}{2l}$, $\curvearrowright$

Studera D:

$a_D = x\alpha$

$\therefore$ $a_D > g$ om $x > \frac{2l}{3}$