

Lektion 1

Uppgifter: 1.11, 1.12,
4a, (4b, 5)

1.11)

a) $\frac{1}{s^2 + s} = \frac{1}{s(s+1)} \xrightarrow{\mathcal{L}^{-1}} \frac{e^{-t} - 1}{0-1} =$
 $= 1 - e^{-t} \rightarrow 1 \text{ då } t \rightarrow \infty$

Från boken

A.20 $\frac{1}{(s+a)(s+b)} \xrightarrow{\mathcal{L}^{-1}} \frac{e^{-bt} - e^{-at}}{a-b}$

En av de viktigaste
när det gäller
Laplace-transform

$$\dot{y} \xrightarrow{\mathcal{L}} sY(s) - y(0)$$

$$b) F(s) = \frac{1}{s^2 - 1} = \frac{1}{(s+1)(s-1)} \xrightarrow{\mathcal{L}^{-1}} \left/ \begin{matrix} a=1 \\ b=-1 \end{matrix} \right/ \xrightarrow{\mathcal{L}^{-1}} \frac{1}{1-(-1)} (e^{-t} - e^t) =$$

$$= \frac{1}{2} (e^{-t} - e^t) \rightarrow \infty, \text{ da } t \rightarrow \infty$$

$$c) F(s) = \frac{1}{(s+1)^2} \xrightarrow{\mathcal{L}^{-1}} \left/ \begin{matrix} a=1 \end{matrix} \right/ \xrightarrow{\mathcal{L}^{-1}} t e^{-t}, \rightarrow 0, \text{ da } t \rightarrow \infty$$

$$1.12) \quad \dot{y}(t) + y(t) = z(t) \quad \textcircled{2}$$

z : inflow

$$\ddot{z}(t) + \dot{z}(t) + z(t) = u(t) \quad \textcircled{1}$$

u : Kontrollsignal

$$\dot{z}(t) = \ddot{y}(t) + \dot{y}(t) \quad \textcircled{3}$$

$$\ddot{z}(t) = \ddot{y}(t) + \dot{y}(t) \quad \textcircled{4}$$

Sätt in $\textcircled{2}, \textcircled{3}, \textcircled{4}$ i $\textcircled{1}$

$$u(t) = \ddot{y}(t) + \dot{y}(t) + \ddot{y}(t) + \dot{y}(t) + \dot{y}(t) + y(t) \Leftrightarrow$$

$$\boxed{\Leftrightarrow u(t) = \ddot{y}(t) + 2\dot{y}(t) + 2y(t)}$$

4)

$$a) \frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = \ddot{y}(t) + 3\dot{y}(t) + 2y(t) = \sigma(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

$$\left. \begin{array}{l} \ddot{y} \xrightarrow{\mathcal{L}} s^2 Y(s) \\ \dot{y} \xrightarrow{\mathcal{L}} s Y(s) \\ y \xrightarrow{\mathcal{L}} Y(s) \end{array} \right\} \begin{array}{l} \downarrow \\ s^2 Y(s) + 3s Y(s) + 2Y(s) = \\ = \sum(s) = \frac{1}{s} \end{array} \xrightarrow{\mathcal{L}^{-1}} G \rightarrow \sigma(t)$$

$$\Leftrightarrow Y(s)(s^2 + 3s + 2) = \frac{1}{s} \Leftrightarrow$$

$$\Leftrightarrow Y(s) = \frac{1}{s(s^2 + 3s + 2)} = \frac{1}{s(s+1)(s+2)}$$

$$\xrightarrow{\mathcal{L}^{-1}} \left/ \begin{array}{l} a=1 \\ b=2 \end{array} \right/ \xrightarrow{\mathcal{L}^{-1}} \frac{1}{2} \left(1 + \frac{1}{1-2} (2e^{-t} - e^{-2t}) \right) =$$

$$= \frac{1}{2} \left(1 - (2e^{-t} - e^{-2t}) \right) = \frac{1}{2} - \frac{1}{2} (2e^{-t} - e^{-2t}) =$$

$$= \frac{1}{2} e^{-2t} - e^{-t} + \frac{1}{2} = y(t), \quad t \geq 0$$

$$y(t) = \frac{1}{2} e^{-2t} - e^{-t} + \frac{1}{2}, \quad t \geq 0$$

$$b) \quad \dot{y}(t) + y(t) = u(t)$$

$$u(t) = 1 + \sin(t)$$

$$y(0) = 0$$

$$\dot{y}(t) + y(t) \xrightarrow{\text{d}} sY(s) + Y(s) = Y(s)(s+1) = U(s) =$$

$$= \frac{1}{s} + \frac{1}{s^2+1} \Leftrightarrow$$

$$\Leftrightarrow Y(s) = \frac{1}{s(s+1)} + \frac{1}{(s+1)(s^2+1)} = (*)$$

$$\frac{1}{(s+1)(s^2+1)} = // \text{Part. brak. uppdel} // = \frac{As+B}{s^2+1} + \frac{C}{s+1} \Leftrightarrow$$

$$\Leftrightarrow 1 = \frac{(As+B)(s+1)(s^2+1)}{s^2+1} + \frac{C(s+1)(s^2+1)}{s+1} = (As+B)(s+1) + C(s^2+1) =$$

$$= As^2 + As + Bs + B + Cs^2 + C = s^2(A+C) + s(A+B) + (B+C) = 1$$

$$\begin{aligned} A+C &= 0 \Leftrightarrow A = -C \\ A+B &= 0 \Leftrightarrow A = -B \end{aligned} \left. \begin{array}{l} \\ \end{array} \right\} B=C$$

$$B+C = B+B = 1 \Leftrightarrow B=C=\frac{1}{2}, \quad A=-B=-\frac{1}{2}$$

$$\Rightarrow \begin{cases} A = -\frac{1}{2} \\ B = \frac{1}{2} \\ C = \frac{1}{2} \end{cases}$$

$$\frac{As+B}{s^2+1} + \frac{C}{s+1} = \frac{-\frac{1}{2}s + \frac{1}{2}}{s^2+1} + \frac{\frac{1}{2}}{s+1} = \frac{\frac{1-s}{2}}{\frac{s^2+1}{1}} + \frac{\frac{1}{2}}{\frac{s+1}{1}} = \frac{1-s}{2(s^2+1)} + \frac{\frac{1}{2}}{2(s+1)}$$

$$(*) = \frac{1}{s(s+1)} + \frac{1-s}{2(s^2+1)} + \frac{1}{2(s+1)} = \frac{1}{s(s+1)} + \frac{1}{2(s^2+1)} - \frac{s}{2(s^2+1)} + \frac{1}{2(s+1)} =$$

$$= \underbrace{\frac{1}{s(s+1)}}_{a=1} + \frac{1}{2} \left(\underbrace{\frac{1}{s^2+1}}_{w_0=1} - \underbrace{\frac{s}{s^2+1}}_{w_0=1} + \underbrace{\frac{1}{s+1}}_{a=1} \right) \xrightarrow{\text{d}^{-1}} (1-e^{-t}) + \frac{1}{2} \left(\sin(t) - \cos(t) + e^{-t} \right) =$$

$$= 1 - e^{-t} + \frac{1}{2} \sin(t) - \frac{1}{2} \cos(t) + \frac{1}{2} e^{-t} \Leftrightarrow$$

$$y(t) = 1 + \frac{1}{2} \sin(t) - \frac{1}{2} \cos(t) - \frac{1}{2} e^{-t}$$