Föreläsning 11

TMME04 – Mekanik II

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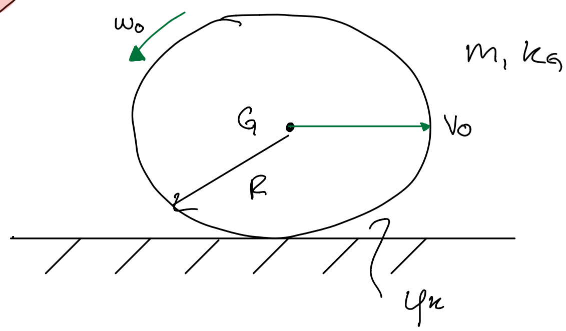
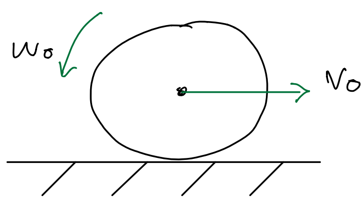


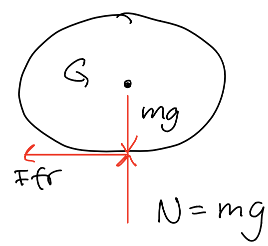
Sökt:

Fart V_{Gs} och tid t_s då slutat spinna.

$t=0$ (spinner)



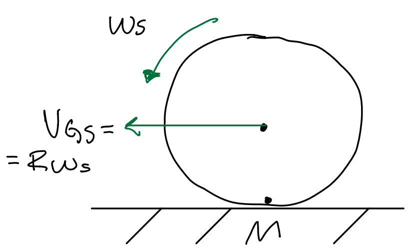
Friläggning, under spinn:



$F_{fr} = \mu_k N = \mu_k mg$

motriktat spinn.

$t = t_s$ (slutat spinna)



Impulslagen

$$\int_{t_1}^{t_2} \bar{F}\,dt = \bar{p}_2 - \bar{p}_1 \,, \quad p = m\bar{v}_G$$

$$\xrightarrow{}: \underbrace{\int_0^{t_s} (-\mu_k m g)\, dt}_{-\mu_k m g t_s} = m(-R\omega_s) - m v_0 \qquad (1)$$

Impulsmoment lagen

$$\int_{t_1}^{t_2} M_G \, dt = (h_G)_2 - (h_G)_1 \,, \quad h_G = I_G \omega$$

$$G: \underbrace{\int_0^{t_s} \mu_k m g \cdot R \, dt}_{\mu_k m g R t_s} = \underbrace{I_G}_{m k_G^2} (-\omega_s - (-\omega_0)) \qquad (2)$$

(1), (2) $\Rightarrow$

$$\omega_s = \frac{k_G^2 \omega_0 - R v_0}{k_G^2 + R^2} \,, \quad V_{Gs} = R\omega_s \,, \leftarrow$$

$$t_s = \frac{k_G^2 (v_0 + R\omega_0)}{\mu_k g (k_G^2 + R^2)}$$

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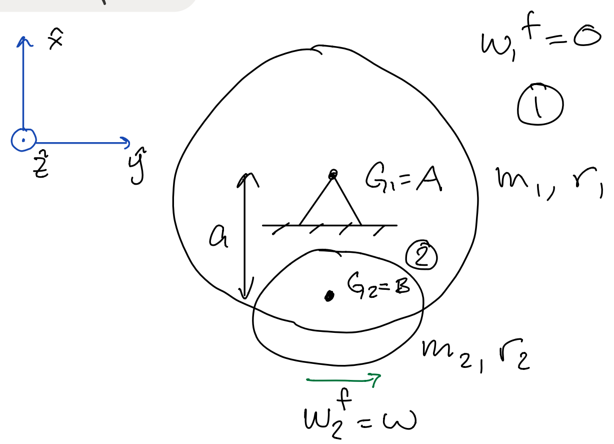
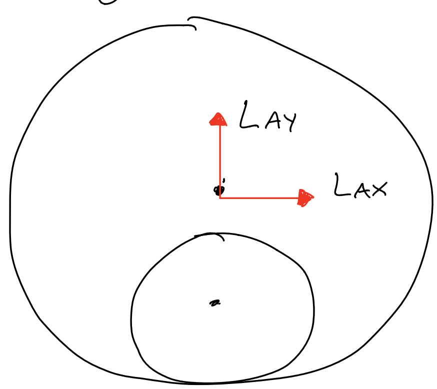
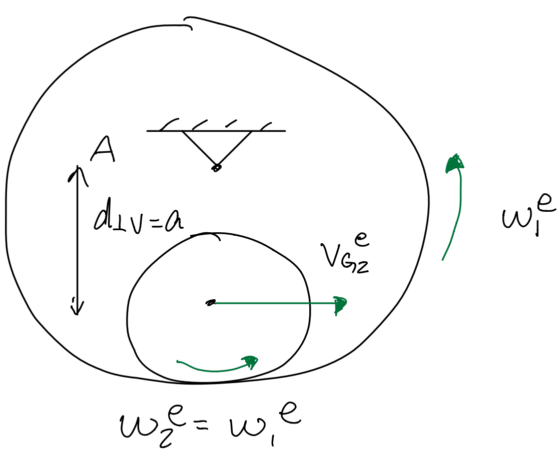
Precis före:



Vid fastlåsning:



Precis efter:



$V_{G_2}^2 = a\omega_1^e$

(cirkelrörelse)

Sökt:

$\omega_1^e = \omega_2^e, \bar{L}_A, \bar{L}_B, \bar{K}_B$

Stötimpulsmomentlagen, för ① + ②

$$A_A^{s,ext} = \sum_{i=1}^{2} h_A^{(i),e} - \sum_{i=1}^{2} h_A^{(i),f},$$

$h_A^{(1)} = I_A^{(1)} \omega$, A fix i i-ram och kropp 1

$h_A^{(2)} = I_{G2}^{(2)} \omega_2 + m_2 v_{G2} d_\perp v$

$$\curvearrowleft A: \quad 0 = \underbrace{I_A^{(1)}}_{\frac{1}{2}m_1 r_1^2} \omega_1^e + \underbrace{I_{G2}^{(2)}}_{\frac{1}{2}m_2 r_2^2} \omega_1^e + m_2 a \omega_1^e a - \left(0 + I_{G2}^{(2)} \omega + 0\right)$$

$$\omega_1^e = \frac{\frac{m_2}{2} r_2^2 \omega}{\frac{m_1}{2} r_1^2 + \frac{m_2}{2} r_2^2 + m_2 a^2}, \curvearrowleft$$

Stötimpulslagen, för ① + ②

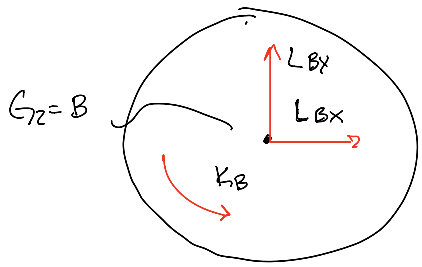
$$\bar{L}^{s,ext} = \sum_{i=1}^{2} \bar{p}^{(i),e} - \sum_{i=1}^{2} \bar{p}^{(i),f}, \quad p^{(i)} = m_i v_{Gi}$$

$\hat{x}: L_{Ax} = 0 - m_2 a \omega_1^e - (0 + 0)$

$\hat{y}: L_{Ay} = 0 - 0$

$\therefore \bar{L}_A = m_2 a \omega_1^e \hat{x}$

Friläggning, 2



Stötimpulslagen

$\hat{x}$: $L_{Bx} = m_2 a \omega_1^e - 0$

$\hat{y}$: $L_{BY} = 0 - 0$

$\therefore \bar{L}_B = m_2 a \omega_1^e \hat{x}$

Stötimpulsmomentlagen

$$A^S_{G_2} = h_{G_2}^{2,e} - h_{G_2}^{2,f}, \quad h_G^{2} = I_{G_2}^{2} \omega_2$$

$\curvearrowleft G_2$: $K_B = I_{G_2}^{2} \omega_1^e - I_{G_2}^{2} \omega$

$$\therefore \bar{K}_B = \frac{m_2}{2} r_2^2 (\omega_1^e - \omega) \hat{z}$$

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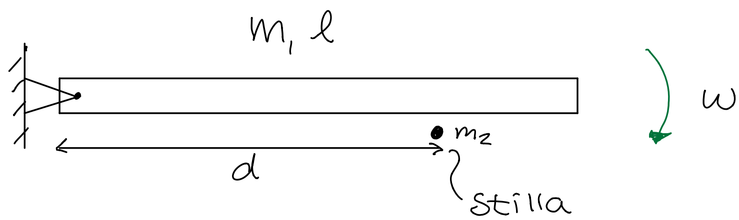


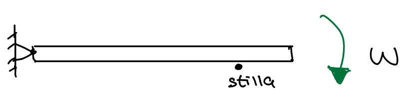
Givet:

Stöttal e

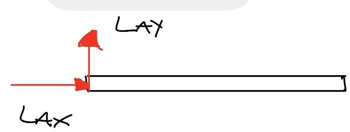
Sökt:

v_2^e och v_2^e maximeras

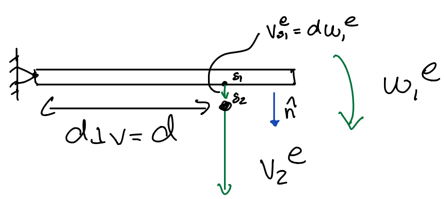
Precis före:



Vid stöt:



Precis efter:



Stötimpulsmomentlagen, för ①+②

$$A_A^{s,ext} = \sum_{i=1}^{2} h_A^{(i),e} - \sum_{i=1}^{2} h_A^{(i),f},$$

$h_A^{(1)} = I_A^{(1)} \omega_1$, A fix i i-ram och kropp 1

$$h_A^{(2)} = \underbrace{I_{G_2}^{(2)}}_{=0} \omega_2 + m_2 V_{G_2} \underbrace{d_{\perp V}}_{d}$$

ty partikel

$\curvearrowright A$: $0 = I_A^{(1)} \omega_1^e + m_2 V_2^e d - (I_A \omega + 0)$ (1)

där

$$I_A^{(1)} = \underbrace{I_{G_1}^{(1)}}_{\frac{m_1 \ell^2}{12}} + m_1 \left(\frac{\ell}{2}\right)^2 = \frac{m_1 \ell^2}{3}$$

Stöttalet,

$$e = \frac{(V_{S_1}^e - V_{S_2}^e) \cdot \hat{n}}{(V_{S_2}^f - V_1^f) \cdot \hat{n}} = \frac{d\omega_1^e - V_2^e}{0 - d\omega} \quad (2)$$

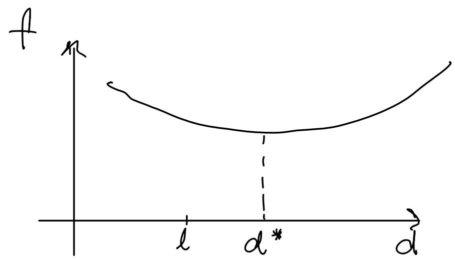
(1), (2) $\Rightarrow$

$$V_2^e = \frac{(1+e) \frac{m_1 \ell^2 \omega}{3}}{\frac{m_1 \ell^2}{3d} + m_2 d}, \downarrow$$

V_2^e är maximal då $f(d) := \frac{m_1 \ell^2}{3d} + m_2 d$ är minimal.

$$f'(d) = -\frac{m_1 \ell^2}{3d^2} + m_2 = 0 \Rightarrow d^* = \sqrt{\frac{m_1}{3m_2}}\, \ell$$

$$f''(d^*) = \frac{2m_1 \ell^2}{3(d^*)^3} > 0$$



$\therefore$ min,

men $d \leq \ell$ så

$d = \min\left(\sqrt{\frac{m_1}{3m_2}}\,\ell, \ell\right) \Rightarrow V_2^e$-max.