Lösningsgång

TANA21 – Beräkningsmatematik

Tenta – 2016-08-22

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① a) 160822

relativt fel 1,2%.

$$\frac{\Delta a}{a} \approx \left|\frac{\Delta a}{\bar{a}}\right| \leq 0{,}0012 = |\Delta a| \leq 1929{,}864$$

190822 − 1929 = 2 iss.

⇒ Signifikanta siffror = 4.

b) Om man subtraherar två lika tal blir felet väldigt stort i förhållande till talen.

c) $\|A\|_1$ Beloppet av varje term.

$$\begin{pmatrix} 1 \\ 5 \\ -4 \end{pmatrix} \Rightarrow \underline{\underline{10}}$$

d) Man sätter ändpunktsvinkler $f'(x)=0$

e) ?

2 a)

$$A = \begin{pmatrix} -0{,}6 & 4 & -0{,}8 \\ -0{,}9 & 1{,}8 & 5{,}9 \\ 3 & 1 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 4{,}8 \\ 129{,}8 \\ 78 \end{pmatrix}$$

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \left(\begin{array}{ccc|c} 3 & 1 & 1 & 78 \\ -0{,}9 & 1{,}8 & 5{,}9 & 129{,}8 \\ -0{,}6 & 4 & -0{,}8 & 4{,}8 \end{array}\right) \begin{array}{l} \rbrack\, 0{,}3\cdot(1) \\ \rbrack\, 0{,}2\cdot(1) \end{array}$$

$$= \left(\begin{array}{ccc|c} 3 & 1 & 1 & 78 \\ 0 & 2{,}1 & 6{,}2 & 153{,}2 \\ 0 & 4{,}2 & -0{,}6 & 20{,}4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 3 & 1 & 1 & 78 \\ 0 & 4{,}2 & -0{,}6 & 20{,}4 \\ 0 & 2{,}1 & 6{,}2 & 153{,}2 \end{array}\right) \rbrack -0{,}5\cdot(2)$$

$$\sim \left(\begin{array}{ccc|c} 3 & 1 & 1 & 78 \\ 0 & 4{,}2 & -0{,}6 & 20{,}4 \\ 0 & 0 & 6{,}5 & 143 \end{array}\right)$$

$$L = \begin{pmatrix} 1 & & \\ -0{,}3 & 1 & \\ -0{,}2 & 0{,}5 & 1 \end{pmatrix} \qquad P = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$U = \begin{pmatrix} 3 & 1 & 1 \\ 0 & 4{,}2 & -0{,}6 \\ 0 & 0 & 6{,}5 \end{pmatrix}$$

b) $Ux = b$

$$\begin{pmatrix} 3 & 1 & 1 \\ 0 & 4{,}2 & -0{,}6 \\ 0 & 0 & 6{,}5 \end{pmatrix} \begin{pmatrix} 78 \\ 20{,}4 \\ 143 \end{pmatrix} \Rightarrow \begin{array}{l} x_3 = 22 \\ x_2 = \dfrac{20{,}4 + 0{,}6\cdot x_3}{4{,}2} = 8 \\ x_1 = \dfrac{78 - 22 - 8}{} = 16 \end{array}$$

3

$x = \begin{pmatrix} 16 \\ 8 \\ 22 \end{pmatrix}$

3)

x	0,35	0,4	0,45	0,5	0,55
$f(x)$	0,37	0,42	0,48	0,55	0,62

$$p(x) = \underset{c_1}{0{,}37}\frac{(x-0{,}4)(x-0{,}45)}{(0{,}35-0{,}4)(0{,}35-0{,}45)} + \underset{c_2}{0{,}42}\frac{(x-0{,}35)(x-0{,}45)}{(0{,}4-0{,}35)(0{,}4-0{,}45)}$$

$$+ \underset{c_3}{0{,}48}\frac{(x-0{,}35)(x-0{,}40)}{(0{,}45-0{,}35)(0{,}45-0{,}4)} = 0{,}4428$$

med Lagrange

b) $p(x) = c_1 + c_2(x-0{,}45) + c_3(x-0{,}45)^2$

$$p(0{,}35) = c_1 + c_2(-0{,}1) + c_3(0{,}1)^2 = 0{,}37$$
$$p(0{,}4) = c_1 + c_2(-0{,}05) + c_3(0{,}05)^2 = 0{,}42$$
$$p(0{,}45) = c_1 + 0 + 0 = 0{,}48$$
$$p(0{,}5) = c_1 + c_2\, 0{,}05 + c_3\, 0{,}025 = 0{,}55$$
$$p(0{,}55) = c_1 + c_2\, 0{,}1 + c_3\, 0{,}01 = 0{,}62$$

$$A = \begin{pmatrix} 1 & -0{,}1 & 0{,}01 \\ 1 & -0{,}05 & 0{,}025 \\ 1 & 0 & 0 \\ 1 & 0{,}05 & 0{,}025 \\ 1 & 0{,}1 & 0{,}01 \end{pmatrix} \qquad A^T = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ -0{,}1 & -0{,}05 & 0 & 0{,}05 & 0{,}1 \\ 0{,}01 & 0{,}0025 & 0 & 0{,}0025 & 0{,}01 \end{pmatrix}$$

$$b = \begin{pmatrix} 0{,}37 \\ 0{,}42 \\ 0{,}48 \\ 0{,}55 \\ 0{,}62 \end{pmatrix} \qquad A^T b = \begin{pmatrix} 2{,}44 \\ 0{,}0315 \\ 0{,}012325 \end{pmatrix}$$

$$A^T A = \begin{pmatrix} 5 & 0 & 0{,}025 \\ 0 & 0{,}025 & 0 \\ 0{,}025 & 0 & 2{,}125 \cdot 10^{-4} \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 5 & 0 & 0{,}025 & 2{,}44 \\ 0 & 0{,}025 & 0 & 0{,}0315 \\ 0{,}025 & 0 & 2{,}125 \cdot 10^{-4} & 0{,}012325 \end{array}\right) \cdot (-0{,}005)$$

$$\sim \left(\begin{array}{ccc|c} 5 & 0 & 0{,}025 & 2{,}44 \\ 0 & 0{,}025 & 0 & 0{,}0315 \\ 0 & 0 & 8{,}75 \cdot 10^{-5} & 1{,}25 \cdot 10^{-4} \end{array}\right)$$

$c_3 = 1{,}428\ldots$

$c_2 = 1{,}26$

$c_1 = 0{,}480$

$\underline{p(0{,}42) = 0{,}44}\ldots$

c) $\displaystyle\int_{0{,}35}^{0{,}55} f(x)\,dx \qquad h = 0{,}05$

$$\frac{h}{3}\left(f(0{,}35)+4f(0{,}4)+2f(0{,}45)+4f(0{,}5)+f(0{,}55)\right)$$

$$=\frac{0{,}05}{3}\left(0{,}37+4\cdot 0{,}42+2\cdot 0{,}48+4\cdot 0{,}55+0{,}62\right)=0{,}0971\ldots$$

Trunkeringsfel:

$$\frac{0{,}05}{3}\cdot(12\cdot 0{,}005)=\underline{\underline{0{,}001}}$$

d) $$f'''=\frac{-f(0{,}35)+2f(0{,}4)-2f(0{,}5)+f(0{,}55)}{2\cdot(0{,}05)^3}$$

$$=\frac{-0{,}37+2\cdot 0{,}42-2\cdot 0{,}55+0{,}62}{2\cdot(0{,}05)^3}=-40$$

fel: 0,005 $\Rightarrow$ avrundningsfelet: $\frac{6\cdot 0{,}005}{2\cdot(0{,}05)^3}=\underline{120}$

$$|\Delta s|\leq\frac{h}{3}|\Delta f_0|+4|\Delta f_1|+\ldots+|\Delta f_4|\leq$$

4 $y''=xy'-y \quad y(0)=1, \; y'(0)=2$

$h=0{,}1 \quad \underline{y(0{,}3)}$

$y_{i+1}=y_i+h\cdot f(x_i, y_i)$

$$\begin{matrix} u=y & u(0)=1 \\ v=y' & v(0)=2 \end{matrix} \Rightarrow Y_0=\begin{pmatrix}1\\2\end{pmatrix}=\begin{pmatrix}u_0\\v_0\end{pmatrix}$$

$$\Rightarrow \begin{cases} u'=v \\ v'=xv-u \end{cases}$$

$$u_{i+1} = \begin{pmatrix} u \\ v \end{pmatrix} + h_i f(x_i \; v_i \; \overset{v_\phi}{} - u_i)$$

$$\Rightarrow \begin{pmatrix} u_1 \\ v_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + 0{,}1 \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 1{,}2 \\ 1{,}9 \end{pmatrix} = ($$

$$\begin{pmatrix} u_2 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1{,}2 \\ 1{,}9 \end{pmatrix} + 0{,}1 \begin{pmatrix} 1{,}9 \\ 0{,}1 \cdot 1{,}9 - 1{,}2 \end{pmatrix} = \begin{pmatrix} 1{,}39 \\ 1{,}799 \end{pmatrix} \quad x = 0{,}1$$

$$\begin{pmatrix} u_3 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1{,}39 \\ 1{,}799 \end{pmatrix} + 0{,}1 \begin{pmatrix} 1{,}799 \\ 0{,}2 \cdot 1{,}799 - 1{,}39 \end{pmatrix} = \begin{pmatrix} 1{,}5699 \\ 1{,}79598 \end{pmatrix}$$

Svar: $y(0{,}3) \approx \underline{\underline{1{,}5699}}$

(5) $e^x - x^3 = 0 \qquad f'x = e^x - 3x^2$

Rot nära 1,9

Newton-Raphsons

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$X = 3$

$$x_{i+1} = 3 - \frac{(e^x - x^3)}{e^x - 3x^2} = 3 - \frac{e^3 - 27}{e^3 - 27} = 2$$

$$x_{i+1} = 2 - \frac{e^2 - 8}{e^2 - 12} = 1{,}867\ldots$$

$$x_{i+1} = 1{,}867\ldots - \frac{e^{1{,}867} - (1{,}867)^3}{e^{1{,}867} - 3 \cdot (1{,}867)^2} \approx 1{,}857$$

$x_{i+1} = 1{,}857 \; 1.8$

$$|\bar{x}-x|^{*} = \frac{|f(\bar{x})|}{|f'(\xi)|} \approx \frac{|f(\bar{x})|}{|f'(\bar{x})|} \leq 2{,}4 \cdot 10^{-9}$$

6 a) Eftersom få steg tas och då blir approximationen dålig. Instabil vid små h. och större differen.

b) Ansatt $y_n(h) \approx C_0 + C_p h^p \Rightarrow 2^p$

$$\frac{y(4h) - y(2h)}{y(2h) - y(h)} \approx 9{,}52 \text{ dvs går mot } 8 \Rightarrow \underline{\underline{p=3}}$$

c) $2^p \Rightarrow p = 1$ ty $t(2n)/t(n)$

Testa fler!

7 a) Dålig, ty 10^{-17} mycket mindre än 2. Får stor multiplikator.

$$A = \begin{pmatrix} 10^{-17} & 1 \\ 2 & 7 \end{pmatrix}$$

b) $K_\infty(A) \approx 43{,}3$

konditionstal

$$\Delta b = \begin{pmatrix} 0{,}1 \\ 0{,}1 \\ 0{,}1 \\ 0{,}1 \end{pmatrix} \quad \|b\|_\infty = 2 \quad \|\Delta b\|_\infty = 0{,}1$$

Konditionstalet ej så stort så lösning rimlig.

$$\frac{\|\Delta x\|_\infty}{\|x\|_\infty} \leq \|A\|_\infty \cdot \|A^{-1}\|_\infty \cdot \frac{\|\Delta b\|_\infty}{\|b\|_\infty} = K_\infty(A) \frac{\|\Delta b\|_\infty}{\|b\|_\infty}$$

$\leq 43{,}3 \cdot \frac{0{,}1}{2} \leq 2{,}165$

Felet i lösningen är mindre.

$$\frac{\|\Delta x\|}{\|x\|} \leq \frac{0{,}4053}{4{,}7065}$$

8e) Euler bakåt

$y_{i+1} = y_i + hf(x_{i+1}, y_{i+1})$

$x_{i+1} = x_i + h$

$y_i \approx y(x_i) \quad y_{i+1} \approx y(x_{i+1})$

$l_i = y_{i+1} - y_i - hf(x_{i+1}, y_{i+1})$

$\Rightarrow l_i = y(x_{i+1}) - y(x_i) - hf(x_{i+1}, y(x_{i+1}))$

$\Rightarrow l_i = y(x_{i+1}) - y(x_{i+1} - h) - hf(x_{i+1}, y(x_{i+1}))$

$\Rightarrow l_i = \cancel{y(x_{i+1})} - (\cancel{y(x_{i+1})} - hy'(x_{i+1}) + \frac{(-h)^2}{2!}y''(x_{i+1}) + \ldots)$

$- hf(x_{i+1}, y(x_{i+1})) =$

$= \underbrace{-h(y'(x_{i+1}) + f(x_{i+1}, y(x_{i+1})))}_{=0} - \frac{h^2}{2}y''(x_{i+1}) + \ldots$

$= -\frac{h^2}{2}y''(x_{i+1}) + \ldots = O(h^2)$

Euler Arameit:

$y_i \approx y(x_i) \quad y_{i+1} \approx y(x_{i+1})$

$y_{i+1} = y_i + h_i f(x_i, y_i) \implies$

$l_i = y_{i+1} - y_i - h_i f(x_i, y_i)$

$\Rightarrow l_i = y(x_i + h) - y(x_i) - h_i f(x_i, y(x_i))$

$\Rightarrow l_i = \cancel{y(x_i)} + h_i y'(x_i) + \frac{h_i^2}{2!} y''(x_i) + \dots - \cancel{y(x_i)} - h_i f(x_i, y(x_i))$

$\Leftrightarrow l_i = h_i \big(\underbrace{y'(x_i) - f(x_i, y(x_i))}_{=0}\big) + \frac{h_i^2}{2!} y''(x_i) + \dots$

$= O(h^2)$.

$y(x_i + (-h_i))$

	n	siftrer
Addition	n	
Div		1
Multi		1
Koeff	n	-1
Summa	2n	+ 1

$\frac{h}{3}\left(f(x_0) + 4 \cdot f(x_1) + f(x_2)\right)$

$n = 2$