Lektion 2

TAMS24 – Statistisk teori

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10.4

$x_1, x_2, \ldots, x_{10}$

$y_1, y_2, \ldots, y_5$

$\bar{x} = 5313, \quad S_x = 5.2$

$\bar{y} = 5309, \quad S_y = 3.0$

Sökt: Medelvärde

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n=10} x_i = \frac{x_1 + x_2 + \ldots + x_{10}}{10} = 5313 \Rightarrow x_1 + \ldots + x_{10} = 53130$$

$$\bar{y} = \frac{1}{n} \sum_{i=1}^{n=5} y_i = \frac{y_1 + \ldots + y_2}{5} = 26545$$

$$\bar{z} = \frac{1}{15} \sum_{i=1}^{15} z_i = \frac{5313 + 26545}{15} \approx 5312$$

från FL

$$S_x^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 = \frac{1}{n-1} \left(\sum_{i=1}^{n} x_i^2 - \underline{n\bar{x}^2} \right)$$

$$\Leftrightarrow \quad n\bar{x}^2 + (n-1) S_x^2 = \sum_{i=1}^{n} x_i^2 \doteq \underline{2.8228 \cdot 10^8} \quad n=10$$

På samma sätt

$$S_y^2 = 1.\underline{40927405} \cdot 10^8$$

$$\sum_{i=1}^{n+m} w_i^2 = 4.23207 \cdot 10^8$$

$$S_w^2 = \frac{1}{n+m-1}\left(\sum_{i=1}^{n+m} w_i^2 - (n+m)\bar{w}^2\right) =$$

$$= \frac{1}{14}\left(4.232 \cdot 10^8 - 15 \cdot (5311{,}66)^2\right) = 23.764$$

$S_w = 4.87$,

Lektionsgenomgång

1. $\mu \approx \hat{\mu} = \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, Note: $\sum_{i=1}^{n} x_i = n x_i$

2. $\sigma^2 \approx \hat{\sigma}^2 = \begin{cases} \text{if } \mu \text{ is known}, \ \sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2 \\ \text{if } \mu \text{ is unknown}, \ \hat{\sigma}^2 = s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \end{cases}$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n} x_i^2 - n x_i^2$$

Note:

$$\sum_{i=1}^{n} x_i^2 = (n-1)s^2 + n\bar{x}^2$$

3. Unbiased: If

$E(\theta) = \theta$, $\hat{H}$ is unbiased

Effective If

$V(\hat{H}_1) < V(\hat{H}_2)$, $\hat{H}$ is more effective than $\hat{H}_2$

Ex 1:

$\mathbb{X}$ with $f_X(x) = \begin{cases} \frac{1}{1-2\theta}, & 2\theta \le x \le 1 \\ 0, & o.w \end{cases}$ $\{x_1, \dots, x_n\}$

Find $\hat{\theta}$ by M.M

Solution:

$$E(\mathbb{X}) = \bar{x}$$

$$\int_{2\theta}^{1} x \cdot \frac{1}{1-2\theta}\, dx = \bar{x}$$

$$\frac{1+2\theta}{2} = \bar{x} \Rightarrow \hat{\theta}_{MM} = (2\bar{x}-1)/2$$

Ex 2:

$\mathbb{X} \sim N(\mu, \sigma)$: $x_1, \dots, x_n$

$\left.\begin{array}{l} \hat{\mu}_1 = 2x_1 \\ \hat{\mu}_2 = X_n - 10 \end{array}\right\}$ point of μ, are they unbiased? if not, adjust it!

Solution

$$E(\hat{\mu}_1) = E(2\mathbb{X}_1) = 2E(\mathbb{X}_1) = 2\mu \neq \mu. \text{ Not!}$$

Adjusted:

$$\hat{\mu}_1 = \frac{2\mathbb{X}_1}{2} = \mathbb{X}_1, \quad \hat{\mu}_1 = x_1$$

$$E(\hat{\mu}_2) = E(\mathbb{X}_n - 10) = \mu - 10 \neq \mu, \text{ not!}$$

Adjusted:

$$\hat{\mu}_2 = \bar{X}_n - 10 + 10 = \bar{X}_n \quad ; \quad \hat{\mu}_2 = x_n$$

11.8

θ^*_{obs} och $\hat{\theta}_{obs}$, oberoende väntevärdesriktiga punktskattning av θ med kända variansen σ_1^2, σ_2^2

a) $\tilde{\theta}_{obs} = a\theta^*_{obs} + (1-a)\hat{\theta}_{obs}$

$$E(\theta^*) = \theta, \quad E(\hat{\theta}) = \theta$$

$$E(\tilde{\theta}) = a\theta^* + (1-a)\hat{\theta} = aE(\theta^*) + (1-a)E(\hat{\theta}) = \theta \blacksquare$$

b) $V(\tilde{\theta}) = V(a\theta^* + (1-a)\hat{\theta}) = \{\text{oberoende}\} =$

$$= a^2 \underbrace{V(\theta^*)}_{\sigma_1^2} + (1-a)^2 \underbrace{V(\hat{\theta})}_{\sigma_2^2} =$$

$$= a^2\sigma_1^2 + (1-a)^2\sigma_2^2 = g(a)$$

$$\frac{d(g(a))}{da} = 2a\sigma_1^2 - 2(1-a)\sigma_2^2 = 0 \Leftrightarrow$$

$$\Leftrightarrow 2(a\sigma_1^2 - \sigma_2^2 + a\sigma_2^2) = 2(a(\sigma_1^2 + \sigma_2^2) - \sigma_2^2) = 0$$

$$\Leftrightarrow a = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$$

$$\tilde{\theta}_{obs} = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}\theta^*_{obs} + \left(1 - \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}\right)\hat{\theta}_{obs}$$

(11.9) Mätningar:

$$\underset{x_1}{1456.3}, \quad \underset{x_2}{1458.5}, \quad \underset{x_3}{1457.7}, \quad \underset{x_4}{1457.2}$$

$X \sim N(\mu, \sigma)$

Sökt: Väntevärdesriktig skattning av variansen σ^2

a) $\mu = 1457.0$

Känd så kan skattas med

$$(\hat{\sigma}_{obs})^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2 =$$

$$= \frac{1}{4}\left((\underset{x_1}{1456.3} - 1457)^2 + (\underset{x_2}{1458.5} - 1457)^2 + (\underset{x_3}{1457.7} - 1457)^2 + \right.$$

$$\left. + (\underset{x_4}{1457.2} - 1457)^2\right) = \frac{1}{4}(0.49 + 2.25 \quad + 0.49 + 0.04) =$$

$$= \underline{\underline{0.8175}}$$

b) μ är okänd.

$$(\hat{\sigma}_{obs})^2 = s^2 = \frac{1}{n-1}\sum_{n=1}^{n}(x_i - \bar{x})^2 = \frac{1}{n-1}\sum_{n=1}^{n} x_i^2 - n\bar{x}^2 =$$

$$= \frac{1}{3}\left((1456.3)^2 + (1458.5)^2 + 1457.7^2 + 1457.2^2 - 4\bar{x}^2)\right) =$$

$$= / \ \bar{x} = \frac{1}{n}\sum_{i=1}^{4} x_i = \frac{1}{4}(1456.3 + 1458.5 + 1457.7 + 1457.2) =$$

$$= 1457{,}425 \ / = \frac{1}{3}(\ldots) = 0.849\ldots \approx 0.85$$

5829,7

c) Nej! Då endast $E(X_i)$ och $V(X_i)$ har använts ovan

1.

Givet:

Oberoende livslängd, exp-delad

$\mu = 5$

En andel a med

$\mu = 1$

$$f(x) = \frac{1}{5} e^{-x/5} (1-a) + e^{-x} a, \quad x > 0.$$

a) n_1 enheter.

Livslängder: $X_1, \ldots, X_{n_1}$.

Momentmetoden ger