Lektion 7

TAMS24 – Statistisk teori

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Lektionsgenomgång

HT-2

$$TS \begin{cases} \text{fact} \\ H_0 \end{cases} + C \begin{cases} \text{fact} \\ H_0 \end{cases} \quad TS \in C \Rightarrow \text{Reject } H_0.$$

Ex:

$$\left.\begin{array}{l} X \sim N(\mu_1, \sigma) \\ Y \sim N(\mu_2, \sigma) \\ Z \sim N(\mu_3, \sigma) \end{array}\right\} \text{independent} \quad \begin{array}{l} \bar{x} = 18{,}1, \; S_1 = 5, \; n_1 = 9 \\ \bar{y} = 14.6, \; S_2 = 7.2, \; n_2 = 16 \\ \bar{z} = 17.1 \; \; S_3 = 6.3, \; n_3 = 13 \end{array}$$

Test the following hypothesis with $\alpha = 1\%$.

a) $H_0: \mu_1 = 18$, $H_1: \mu_1 < 18$

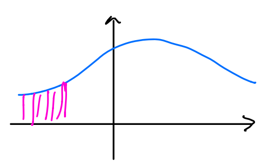
fact:

$$\frac{\bar{X} - \mu}{S/\sqrt{n_1}} \sim t(n_1 + n_2 + n_3 - 3)$$

ty σ unknown

$$S'^2 = \frac{(n_1 - 1)S_1'^2 + (n_2 - 1)S_2'^2 + (n_3 - 1)S_3'^2}{n_1 + n_2 + n_3 - 3}$$

$$TS = \frac{\bar{x} - 18}{S/\sqrt{9}} \approx 0.05$$



$$C = (-\infty, -t_{0.01}(n_1 + n_2 + n_3 - 3)) = (-\infty, -2.44)$$

$TS \notin C \Rightarrow$ don't reject H_0

b) $H_0: \mu_1 - \mu_2 = 0 \qquad H_1: \mu_1 - \mu_2 > 0$

fact:

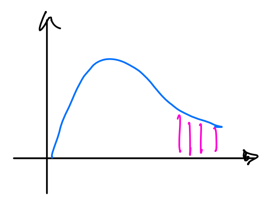
$$\frac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{S' \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t(n_1 + n_2 + n_3 - 3)$$

$$TS = \frac{(\bar{x} - \bar{y}) - 0}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \approx 0.55$$

$$C = \left(t_{0.01}(n_1 + n_2 + n_3 - 3), \ \infty\right) \approx (2.44, \infty), \ TS \notin C$$

$\Rightarrow$ We don't reject H_0.

c) $H_0: \sigma = 6 \qquad H_1: \sigma > 6$



fact: $\frac{(n_1 + n_2 + n_3 - 3) S'^2}{\sigma^2} \sim \chi^2(n_1 + n_2 + n_3 - 3)$

$$TS = \frac{(n_1 + n_2 + n_3 - 3) S'^2}{6^2} \approx 40$$

$$C = \left(\chi^2_{0.01}(n_1 + n_2 + n_3 - 3), \ \infty\right) = \{table\} = (57, \infty)$$

$TS \notin C \Rightarrow$ don't reject H_0.

13.8

$\bar{X} \sim N(\mu, 0.2)$, $\sigma = 0.2$

$H_0: \mu = 4.0$, $H_1: \mu > 4.0$, $\alpha = 0.05$

fact:

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(\mu, 0.2)$$

$$\frac{\bar{x} - 4}{0.2/\sqrt{10}} = \frac{4.1 - 4}{0.2/\sqrt{10}} = 1.581\ldots \qquad 4.1 - 1.581$$

$$C = (\lambda_{0.05}, \infty) = \cancel{\Phi(\lambda) = 0.05 \Rightarrow 1.64} = (1.64, \infty)$$

$TS \notin C \Rightarrow$ förkasta ej H_0.

13.9

$$h(\mu) = P(H_0 \text{ förkastas}, \mu = 4) = P(TS \in C, \mu = 4) =$$

$$= P\left(\frac{\bar{X} - 4}{\sigma/\sqrt{n}} > \frac{4 + \lambda_{0.05} \cdot \frac{\sigma}{\sqrt{n}} - 3.8}{\sigma/\sqrt{n}}\right) = P(N(0,1) > 4.8\ ) =$$

$$= 1 - \Phi(4.8\ ) \approx 1 - 0.999.. \approx 0$$

$$h(\mu) = P(H_0 \text{ förkastas}, \mu = 4) = P(TS \in C, \mu = 4) =$$

$$= P\left(\frac{\bar{X} - 4}{\sigma/\sqrt{n}} > \frac{4 + \lambda_{0.05} \cdot \frac{\sigma}{\sqrt{n}} - 4.3}{\sigma/\sqrt{n}}\right) = P(N(0,1) > -3.10\ldots) =$$

$$= (1 - (1 - \Phi(3.10\ldots)) \approx 0.9990$$

13.11

$X \sim N(\mu, 2)$, $\bar{X} \sim N(0, \frac{2}{\sqrt{n}})$

$H_0: \mu = 1$, $H_1: \mu < 1$, $\alpha = 0.05$

Medelvärdet $\bar{x}$ av n oberoende observationer

förkasta H_0 om

$\bar{x} < 1 - \frac{2\lambda_{0.05}}{\sqrt{n}}$

$H(\mu) = P(H_0$ förkastas om $\bar{x} < 1 - \frac{2\lambda_{0.05}}{\sqrt{n}}, \mu = 0) =$

$P(TS \in C$ om $\bar{x} < 1 - \frac{2\lambda_{0.05}}{\sqrt{n}}, \mu = 0) =$

$= P(\bar{X} - 0 < 1 - \lambda_{0.05} \frac{2}{\sqrt{n}}) = P(\frac{\bar{X} - 0}{2/\sqrt{n}} < \frac{1}{2/\sqrt{n}} - \lambda_{0.05}$

$= \Phi(\frac{1}{2/\sqrt{n}} - \lambda_{0.05})$

$\lambda_{0.01} \leq \frac{1}{2/\sqrt{n}} - \lambda \iff n \geq 4(\lambda_{0.01} + \lambda_{0.05})^2 = 63.088.$

$\Rightarrow$ tag 64 eller fler observationer för att erhålla önskad styrka.

13.12 $H_0: \mu = 1050^\circ C$ $H_1: \mu \neq 1050^\circ C$, $\alpha = 0.05$

$\bar{x} = 1050.92$ $n = 10$

$X \sim N(\mu, \sigma)$

$$s = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 = 2.028$$

Förkastar H_0 för stora värden på $|\bar{x} - \mu_0|$, alternativt $|t|$

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = 1.4345$$

$P(|T| > t_{\alpha/2}) = \alpha$, $\alpha = 0.05$, $t_{\alpha/2} = 2.2622$

Förkastar H_0 om

$|t| > 2.2622$

$\Rightarrow$ Förkastar ej H_0 på nivån $\alpha = 0.05$