

# Lektion 1

TAMS24 – Statistisk teori

Skreven av Oliver Wettergren

[oliwe188@student.liu.se](mailto:oliwe188@student.liu.se)

<https://www.instagram.com/olwettergren/>

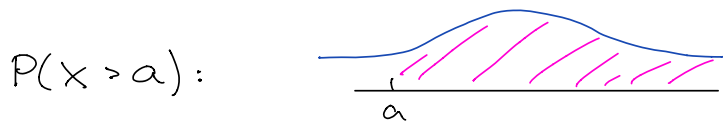
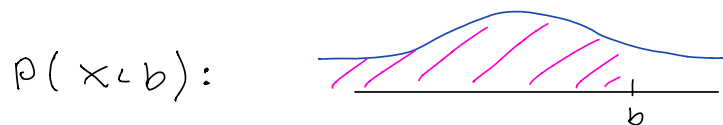
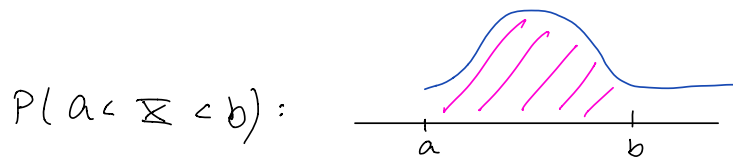
# Lektionsgenömgang

Ex:

$$\left. \begin{aligned} X_1 &\sim N(0, 1) \\ X_2 &\sim N(1, 2) \\ X_3 &\sim N(2, 1) \end{aligned} \right\} \text{independent}$$

$$E(2X_1 - 3X_2 + X_3 - 1) = 2 \overset{0}{E(X_1)} - 3 \overset{1}{E(X_2)} + \overset{2}{E(X_3)} - 1 = -2$$
$$V(2X_1 - 3X_2 + X_3 - 1) = 2^2 \overset{1}{V(X_1)} + (-3)^2 \overset{2}{V(X_2)} + \overset{1}{V(X_3)} = 41$$

försätter du öberoende



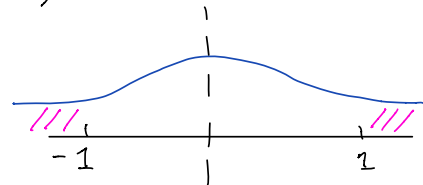
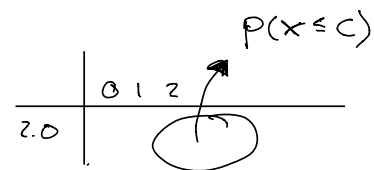
Ex:

$$X \sim N(0, 1)$$

$$P(-1 < X < 2) = P(X < 2) - P(X < -1) =$$

$$= \Phi(2) - (1 - P(X < 1)) =$$

$$= \Phi(2) + \Phi(1) - 1 = 0.9772 - (1 - 0.8413) = 0.8185.$$



$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int e^{kx} dx = \frac{e^{kx}}{k} + C$$

5.7

$$E(e^x)$$

Om

$$f_X(x) = 2e^{-2x}, x \geq 0$$

$$E(e^x) = \int_0^{\infty} 2e^{-2x} \cdot e^x dx = \left[ \frac{2e^{-x}}{-1} \right]_0^{\infty} = 2e^{-0} = \underline{\underline{2}}$$

5.12

$X$ : täthetsfunktion:

$$f_X(x) = 3x^{-4}, x \geq 1$$

$$E(X) = \int_1^{\infty} x \cdot 3x^{-4} dx = \left[ 3 \cdot \frac{x^{-2}}{-2} \right]_1^{\infty} = -\frac{3}{2}$$

$$E(X^2) = \int_1^{\infty} x^2 \cdot 3x^{-4} dx = \left[ \frac{3x^{-1}}{-1} \right]_1^{\infty} = 3$$

$$V(X) = -E(X)^2 + E(X^2) = -\frac{9}{4} + \frac{12}{4} = \frac{3}{4}$$

5.22

$X_1, X_2, X_3$  übereinander

$$\begin{cases} E(X_i) = 2 \\ D(X_i) = 3 \end{cases}, \quad i = 1, 2, 3$$

$$E(Y) = 3E(X_1) - 2E(X_2) + E(X_3) - 6 = 3 \cdot 2 - 2 \cdot 2 + 2 - 6 = -2$$

$$D(Y) = \sqrt{V(Y)} = \sqrt{9V(X_1) + 4V(X_2) + V(X_3)} = \sqrt{9 \cdot 3 + 4 \cdot 3 + 3} = \sqrt{90 + 36} = \underline{\underline{\sqrt{126}}}$$

6.1

$$X \sim N(0, 1)$$

a)  $P(X \leq 1.82) = \overset{\text{tabell}}{\Phi}(1.82) = 0.9656$

b)  $P(X \leq -0.35) = 1 - \Phi(0.35) = 1 - 0.6331 = \underline{\underline{0.3669}}$   
 $\uparrow$   
 $P(X \geq 0.35)$

c)  $P(-1.2 \leq X \leq 0.5) = P(X \leq 0.5) - \overbrace{P(-1.2 \leq X)}^{P(1.2 \geq X)} =$   
 $= \Phi(0.5) - (1 - \Phi(1.2)) =$

d)  $P(X > a) = 0.05 \Leftrightarrow P(X < a) = 0.95 \Rightarrow \underline{\underline{a = 1.65}}$

e)  $P(|X| < a) = 0.95 \Leftrightarrow P(-a \leq X \leq a) = 0.95$

$$\Leftrightarrow P(X < a) - (1 - P(X < a)) = 0.95$$

$$\Leftrightarrow 2P(X < a) = 1.95 \Leftrightarrow P(X \leq a) = 0.975 \Rightarrow a = \underline{\underline{1.96}}$$

5.5