Föreläsning 14

TMME04 – Mekanik II

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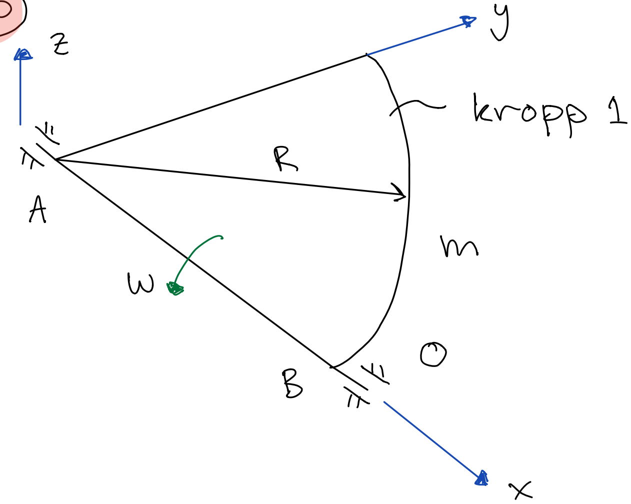
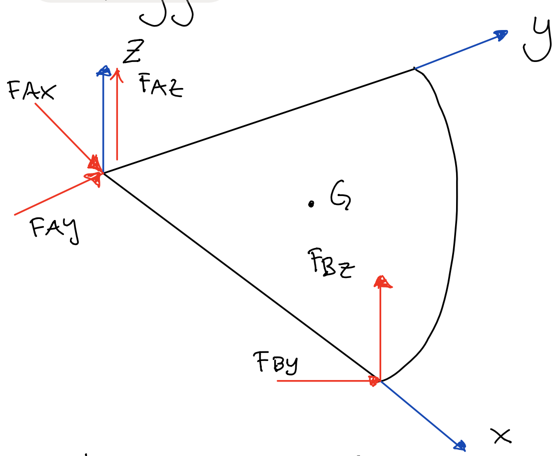


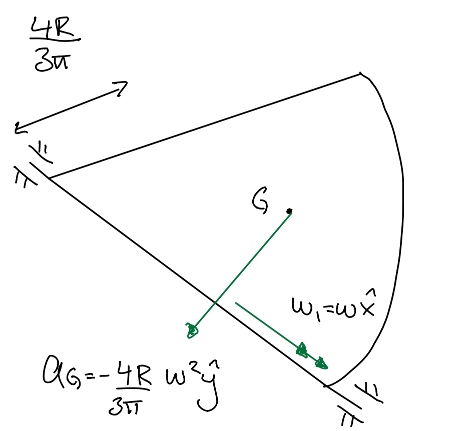
Givet: w konst, Axyz kroppsfix

Sökt: Dynamiska reaktionskrafter $\bar{F}_A$ och $\bar{F}_B$

Frilägg:



Ingen tyngdkraft ty dynamisk



Cirkelrörelse => w konst

Euler I:

$$\bar{F} = m\bar{a}_G$$

$\hat{x}$: $F_{Ax} = 0$ (1)

$\hat{y}$: $F_{Ay} + F_{By} = m a_{Gy} = m\left(-\frac{4R}{3\pi} \omega^2\right)$ (2)

$\hat{z}$: $F_{Az} + F_{Bz} = 0$ (3)

Euler II:

$\bar{M}_A = \dot{\bar{h}}_A$, A fix i i-ram.

$\bar{h}_A = I_A \bar{\omega}_1$, A även fix i kropp 1

$\bar{M}_A = \frac{d\bar{h}_A}{dt} + \bar{\omega}_r \times \bar{h}_A$ (4)

Val av referensram r att derivera i:

$r = 1$

Inför Axyz fixt i kropp 1

$I_{Axz} = \int \underbrace{-xz\,dm}_{=0}$

$$\bar{h}_A = \begin{bmatrix} I_{Axx} & * & * \\ I_{Axy} & * & * \\ 0 & * & * \end{bmatrix} \begin{bmatrix} \omega \\ 0 \\ 0 \end{bmatrix} = I_{Axx}\omega\hat{x} + I_{Axy}\omega\hat{y}$$

$\left(\frac{d\bar{h}_A}{dt}\right)_1 = \bar{0}$

Insättning i (4) $\Rightarrow$

$\bar{M}_A = \omega\hat{x} \times (I_{Axx}\omega\hat{x} + I_{Axy}\omega\hat{y}) = I_{Axy}\omega^2\hat{z}$ (5)

Men $\bar{M}_A$ i V.L ges av friläggningsfiguren

$M_{Ax} = 0 \overset{(5)}{=} 0$

$$M_{Ay} = -F_{Bz} R \overset{(5)}{=} 0 \quad (6)$$

$$M_{Az} = F_{By} R \overset{(5)}{=} I_{Axy} \omega^2 \quad (7)$$

$(1) \Rightarrow$

$$F_{Ax} = 0$$

$(6),(3) \Rightarrow$

$$F_{Az} = F_{Bz} = 0$$

$(7) \Rightarrow$

$$F_{By} = \frac{I_{Axy}}{R} \omega^2 \quad (8)$$

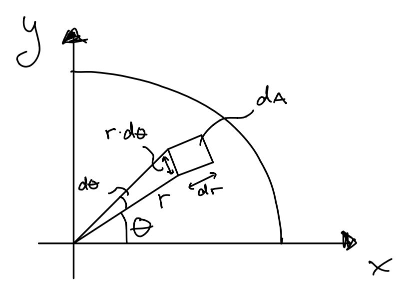
Insättning i $(2) \Rightarrow$

$$F_{Ay} = -\frac{I_{Axy}}{R} \omega^2 - \frac{4mR}{3\pi} \omega^2 \quad (9)$$

I_{Axy}?

$$I_{Axy} = -\int xy \, dm =$$

$$= \Big/ \begin{matrix} x = r\cos\theta, \; y = r\sin\theta \\ dm = m \cdot \frac{dA}{\pi R^2/4} = \frac{4m}{\pi R^2} dr \cdot r d\theta \end{matrix} \Big/ =$$



$$= -\frac{4m}{\pi R^2} \int_0^{\pi/2} \int_0^R r^2 \sin\theta \cos\theta \, dr d\theta = -\frac{2m}{\pi R^2} \cdot \frac{1}{4} R^2 \int_0^{\pi/2} \sin 2\theta \, d\theta =$$

$$= -\frac{mR^2}{4\pi} \Big[-\cos 2\theta \Big]_0^{\pi/2} = \frac{-mR^2}{2\pi}$$

(8) =>

$$F_{By} = -\frac{mR}{2\pi}\omega^2$$

(9) =>

$$F_{Ay} = -\frac{5mR}{6\pi}\omega^2$$

$$\therefore \quad \bar{F}_B = -\frac{mR}{2\pi}\omega^2 \hat{y}$$

$$\bar{F}_A = -\frac{5mR}{6\pi}\omega^2 \hat{y}$$

Ser att F=0 då ω=0, så för alla dynamiska krafter.

Dessutom kraften vid A > B.

Standard: Euler I & II. Tröghetsprodukten skiljer

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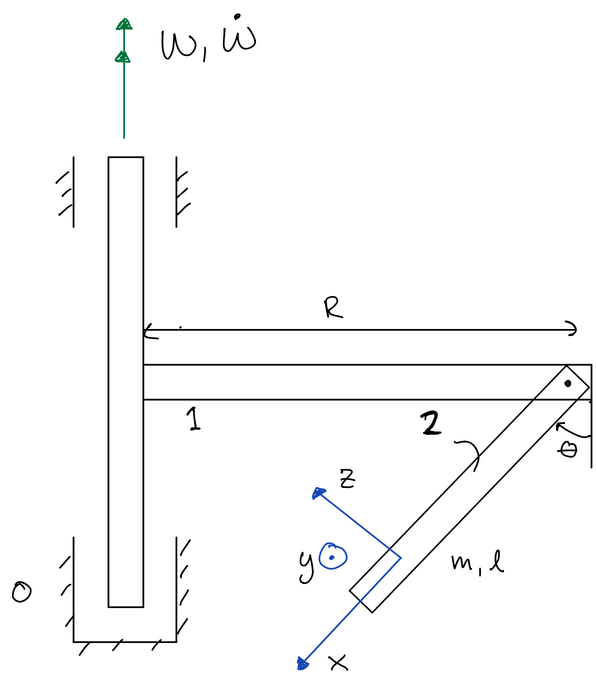
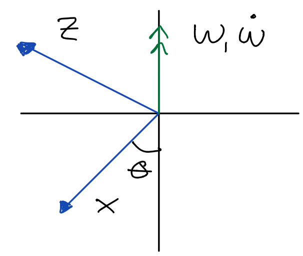


Givet: Gxyz fixt i stången

Sökt: $\bar{\alpha}_2$, T, $\bar{h}_O$



a)

$$\bar{\omega}_2 = \omega_{2/0} = \bar{\omega}_{2/1} + \bar{\omega}_{1/0} = -\dot{\theta}\hat{y} + \omega(-\cos\theta\,\hat{x} + \sin\theta\,\hat{z}) \quad (1)$$

$$\bar{\alpha}_2 = \bar{\alpha}_{2/0} = \left(\frac{d\bar{\omega}_{2/0}}{dt}\right)_0 = \left(\frac{d\bar{\omega}_{2/1}}{dt}\right)_0 + \underbrace{\left(\frac{d\bar{\omega}_{1/0}}{dt}\right)_0}_{\bar{\alpha}_{1/0}} =$$

$$= \underbrace{\left(\frac{d\bar{\omega}_{2/1}}{dt}\right)}_{\bar{\alpha}_{2/1}} + \bar{\omega}_{1/0} \times \bar{\omega}_{2/1} + \bar{\alpha}_{1/0} =$$

$$= -\ddot{\theta}\hat{y} + \omega(-\cos\theta\,\hat{x} + \sin\theta\,\hat{z}) \times (-\dot{\theta}\hat{y}) +$$

$$+ \dot{\omega}(-\cos\theta\,\hat{x} + \sin\theta\,\hat{z}) = (\omega\dot{\theta}\sin\theta - \dot{\omega}\cos\theta)\hat{x} -$$

$$- \dot{\theta}\hat{y} + (\omega\dot{\theta}\cos\theta + \dot{\omega}\sin\theta)\hat{z}$$

Alt:

$$\bar{\alpha}_{2/0} = \left(\frac{d\bar{\omega}_{2/0}}{dt}\right)_0 = \left(\frac{d\bar{\omega}_{2/0}}{dt}\right)_2 + \underbrace{\bar{\omega}_{2/0} \times \bar{\omega}_{2/0}}_{=0} =$$

$$= -\ddot{\theta}\hat{y} + (-\dot{\omega}\cos\theta + \omega\sin\theta\,\dot{\theta})\hat{x} + (\dot{\omega}\sin\theta + \omega\cos\theta\,\dot{\theta})\hat{z}$$

b)

$$T = \frac{1}{2} m \bar{v}_G \cdot \bar{v}_G + \frac{1}{2} \bar{\omega}_z \cdot \bar{h}_G \quad (2)$$

$$\bar{V}_G = \underbrace{\bar{V}_O}_{-R\omega\hat{y}} + \bar{\omega}_z \times \underbrace{\bar{r}_{OG}}_{l/2\,\hat{x}} \overset{(1)}{=} -R\omega\hat{y} + \frac{l}{2}\dot{\theta}\hat{z} + \frac{l}{2}\omega\sin\theta\hat{y} \quad (3)$$

Cirkelrörelse

$$\bar{V}_G \neq \underbrace{\bar{V}_A}_{=0} + \bar{\omega}_z \times \bar{r}_{AG}$$ fungerar ej ty ej samma kropp

$$\bar{h}_G = \bar{I}_G\,\bar{\omega}_z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & ml^2/12 & 0 \\ 0 & 0 & ml^2/12 \end{bmatrix} \begin{bmatrix} -\omega\cos\theta \\ -\dot{\theta} \\ \omega\sin\theta \end{bmatrix} =$$

$$= \frac{ml^2}{12}\dot{\theta}\hat{y} + \frac{ml^2}{12}\omega\sin\theta\hat{z} \quad (4)$$

Insättning i (2) $\Rightarrow$

$$T = \frac{ml^2}{6}\dot{\theta} + \frac{ml^2}{6}\omega^2\sin^2\theta + \frac{mR^2}{2}\omega^2 - \frac{mRl}{2}\omega^2\sin\theta$$

c)

$$\bar{h}_O = \bar{h}_G + \bar{r}_{OG} \times m\bar{v}_G \overset{(3),(4)}{=} -\frac{ml^2}{3}\dot{\theta}\hat{y} +$$

$$+ ml\omega\left(\frac{l}{3}\sin\theta - \frac{R}{2}\right)\hat{z}$$