Föreläsning 3

TMME04 – Mekanik II

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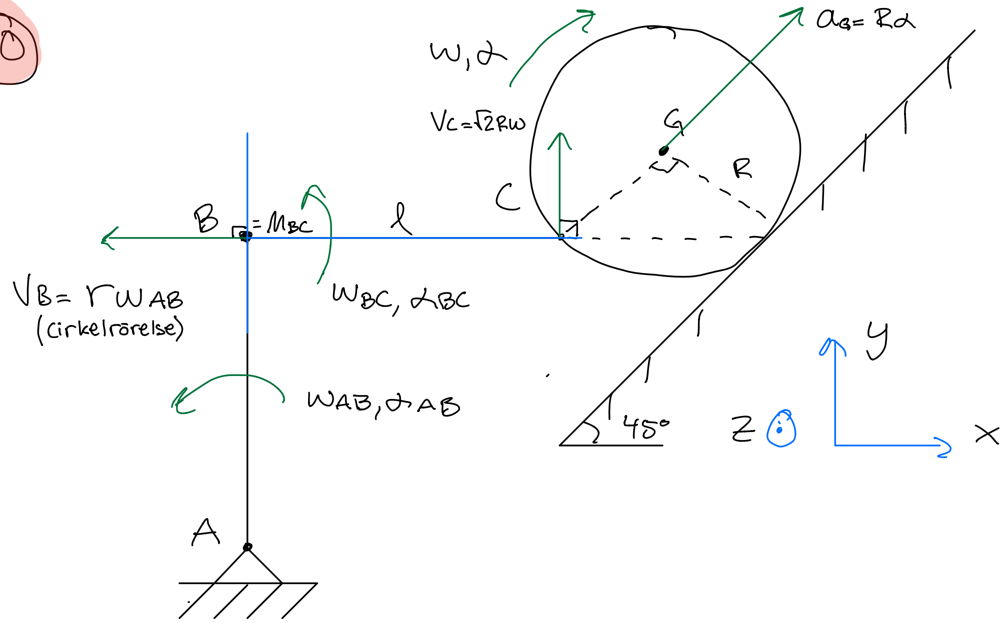


Givet: ω, α, ingen glidning

Sökt: ω_{AB}, ω_{BC}, α_{AB}, α_{BC}

Hastighetsanalys

Stång AB:

$$V_B = r\omega_{AB} \quad (1)$$

Hjulet:

$$V_C = \sqrt{2}R\omega$$

Stång BC:

$$M_{BC} = B \Rightarrow V_B = 0$$

$$\Rightarrow V_C = l\omega_{BC} \Rightarrow \omega_{BC} = \frac{V_C}{l} = \frac{\sqrt{2}R\omega}{l}, \circlearrowleft \quad (2)$$

$(1) \Rightarrow$

$$\omega_{AB} = 0$$

Accelerationsanalys:

Stång AB:

Cirkelrörelse $\Rightarrow$ $\bar{a}_B = -r\alpha_{AB}\hat{x} - r\underbrace{\omega_{AB}^2}_{=0}\hat{y}$

Hjulet:

$$\bar{a}_C = \bar{a}_G + \bar{\alpha} \times \bar{r}_{GC} - \omega^2 \bar{r}_{GC} =$$

$$= \frac{R\alpha}{\sqrt{2}}(\hat{x}+\hat{y}) + \underbrace{(-\alpha\hat{z}) \times \frac{R}{\sqrt{2}}(-\hat{x}-\hat{y})}_{\frac{R\alpha}{\sqrt{2}}(\hat{y}-\hat{x})} - \frac{\omega^2 R}{\sqrt{2}}(-\hat{x}-\hat{y}) \quad (4)$$

Stång BC:

$$\bar{a}_B = \bar{a}_C + \underbrace{\bar{\alpha}_{BC}}_{\alpha_{BC}\hat{z}} \times \underbrace{\bar{r}_{CB}}_{-l\hat{x}} - \omega_{BC}^2 \bar{r}_{CB} \overset{(4),(2)}{=} \sqrt{2}R\alpha\hat{y} +$$

$$+ \frac{R\omega^2}{\sqrt{2}}(\hat{x}+\hat{y}) - l\alpha_{BC}\hat{y} + \frac{2R^2\omega^2}{l}\hat{x} \quad (5)$$

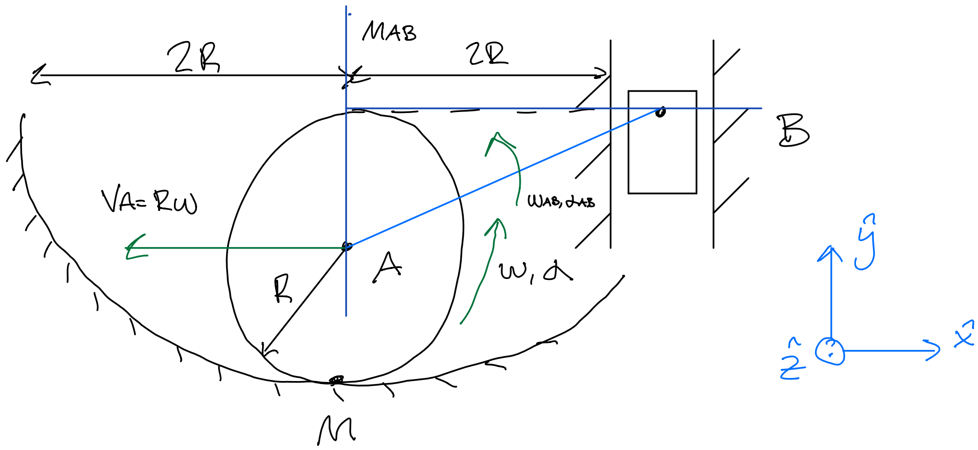
Identifiering, av (5) = (3):

$\hat{x}$: $\frac{R\omega^2}{\sqrt{2}} + \frac{2R^2\omega^2}{l} = -r\alpha_{AB} \Leftrightarrow \alpha_{AB} = \frac{R\omega^2}{\sqrt{2}r} + \frac{2R^2\omega^2}{rl}$, ↻

$\hat{y}$: $\sqrt{2}R\alpha + \frac{R\omega^2}{\sqrt{2}} - l\alpha_{BC} = 0 \Leftrightarrow$

$\Leftrightarrow \alpha_{BC} = \frac{\sqrt{2}R\alpha}{l} + \frac{R\omega^2}{\sqrt{2}l}$, ↺

(11)



Givet: ω, α, ingen glidning

Sökt: ω_{AB}, α_{AB}

Hastighetsanalys

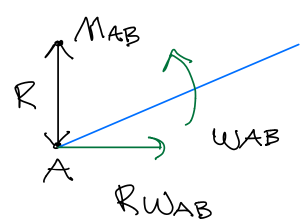
Hjulet:

$$V_A = R\omega$$

Stång AB:

$$V_A = -R\omega_{AB}$$

(OBS!)



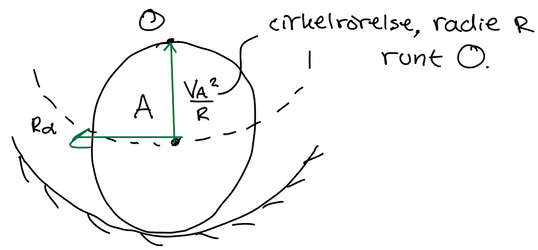
Alt: $\underbrace{\bar{V}_A}_{-R\omega\hat{x}} = \underbrace{\bar{V}_B}_{-V_B\hat{y}} + \bar{\omega}_{AB} \times \bar{r}_{AB}$

$\therefore\ \omega_{AB} = -\omega \Leftrightarrow \Leftrightarrow \omega_{AB} = \omega$, ↻

Accelerationsanalys

Hjulet:

$$\bar{a}_A = -R\alpha\hat{x} + \frac{V_A}{R}\hat{y} =$$
$$= -R\alpha\hat{x} + R\omega^2\hat{y} \quad (1)$$



Stång AB:

$$\bar{a}_A = \bar{a}_B + \bar{\alpha}_{AB} \times \bar{r}_{BA} - \omega_{AB}^2 \bar{r}_{AB} =$$
$$= -a_B\hat{y} + \underbrace{(\alpha_{AB}\hat{z}) \times (-2R\hat{x} - R\hat{y})}_{-2R\alpha_{AB}\hat{y} + R\alpha_{AB}\hat{x}} - (-\omega)^2(-2R\hat{x} - R\hat{y}) \quad (2)$$

Identifiering, av (1) = (2):

$$\hat{x}: -R\alpha = R\alpha_{AB} + 2R\omega^2 \Leftrightarrow \alpha_{AB} = -\alpha - 2\omega^2$$
$$\Leftrightarrow \alpha_{AB} = \alpha + 2\omega^2, \curvearrowright$$

Kunna alla kinematiska tvång i kompendiet!

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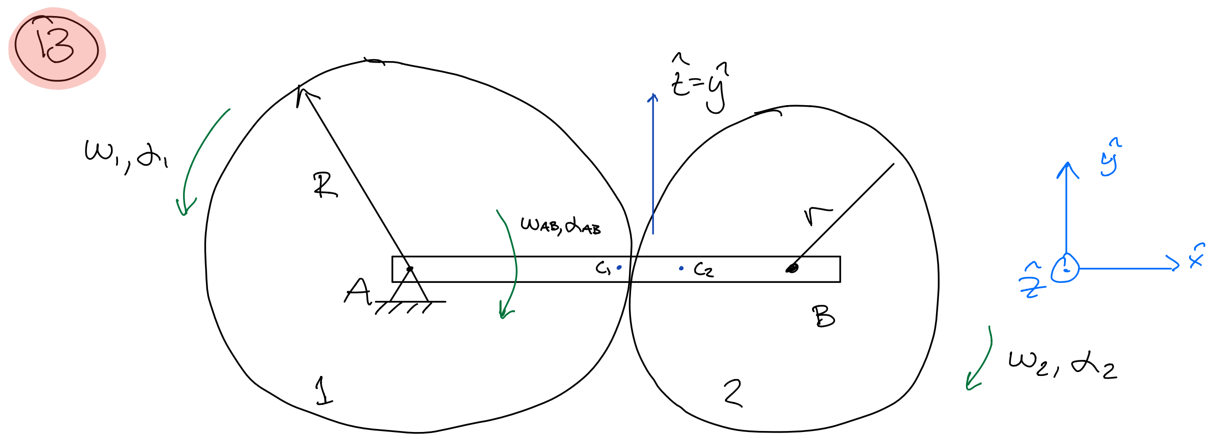
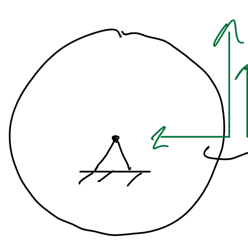


Givet: $\omega, \alpha, \omega_{AB}, \alpha_{AB}$, ingen glidning

Sökt: ω_2, α_2

Hjul 1, cirkelrörelse



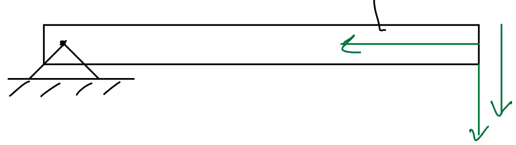
$v_{C_1} = R\omega_1$

$a_{C_1 t} = R\alpha_1$

$a_{C_1 n} = R\omega_1^2$

Stång AB, cirkelrörelse

$a_{Bn} = (R+R)\,\omega_{AB}^2$

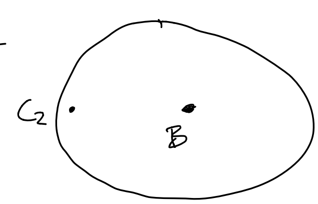


$v_B = (R+r)\,\omega_{AB}$

$a_{Bt} = (R+r)\,\alpha_{AB}$

Hjul 2:

$$\bar{v}_{C_2} = \bar{v}_B + \bar{\omega}_2 \times \bar{r}_{BC_2} = -(R+R)\,\omega_{AB}\hat{y} +$$
$$+ (-\omega_2 \hat{z}) \times (-r\hat{x}) = -(R+r)\,\omega_{AB}\hat{y} +$$
$$+ r\omega_2 \hat{y}$$



Ingen glidning ger:

$$\bar{v}_{C_1} = \bar{v}_{C_2}$$

$$R\omega_1 \hat{y} = -(R+r)\omega_{AB}\hat{y} + r\omega_2 \hat{y}$$

$$\Leftrightarrow \omega_2 = \omega_{AB} + \frac{R}{r}(\omega_1 + \omega_{AB}),\ \curvearrowright$$

$$\bar{a}_{C_2} = \bar{a}_B + \bar{\alpha}_2 \times \bar{r}_{BC_2} - \omega_2^2\,\bar{r}_{BC_2} =$$

$$= -(R+r)\omega_{AB}\hat{x} - (R+r)\alpha_{AB}\hat{y} + \underbrace{(-\alpha_2\hat{z})\times(-r\hat{x})}_{r\alpha_2\hat{y}} -$$

$$- \omega_2^2(-r\hat{x})$$

Ingen glidning ger

$$a_{C_1 t} = a_{C_2 t} : \quad (\hat{t} = \hat{y})$$

$$R\alpha_1 = -(R+r)\alpha_{AB} + r\alpha_2$$

$$\Leftrightarrow \alpha_2 = \alpha_{AB} + \frac{R}{r}(\alpha_1 + \alpha_{AB}),\ \curvearrowright$$

- $\omega_1 = 0,\ R = r \Rightarrow \omega_2 = 2\omega_{AB}$