Föreläsning 6

TMME04 – Mekanik II

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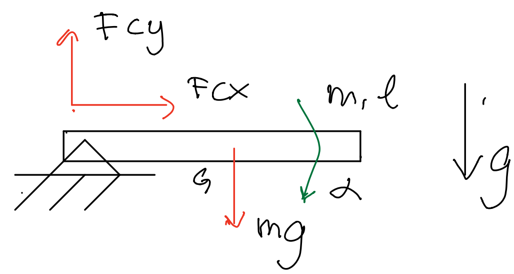
$$\bar{F} = m\bar{a}_G \qquad (11)$$

$$\bar{M}_A = \bar{M}_B + \bar{r}_{AB} \times \bar{F} \qquad (12)$$

$$\bar{M}_A = \dot{\bar{h}}_A + \bar{v}_A \times m\bar{v}_G \qquad (15)$$

$$\bar{M}_G = \dot{\bar{h}}_G \qquad (16)$$

$$\bar{M}_G = I_G \alpha \qquad (18)$$



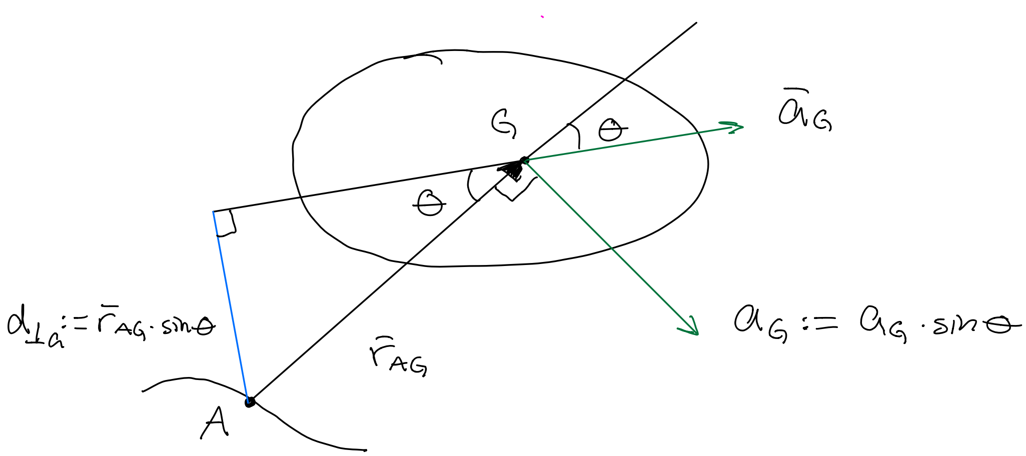
Momentlagen, godtycklig punkt A:

Välj $B = G$ i (12)

$$\bar{M}_A = \underbrace{\bar{M}_G}_{\dot{\bar{h}}_G = I_G \bar{\alpha}} + \bar{r}_{AG} \times \underbrace{\bar{F}}_{m\bar{a}_G}$$

$$\boxed{\bar{M}_A = I_G \bar{\alpha} + \bar{r}_{AG} \times m\bar{a}_G} \qquad (19)$$

Skalärt:



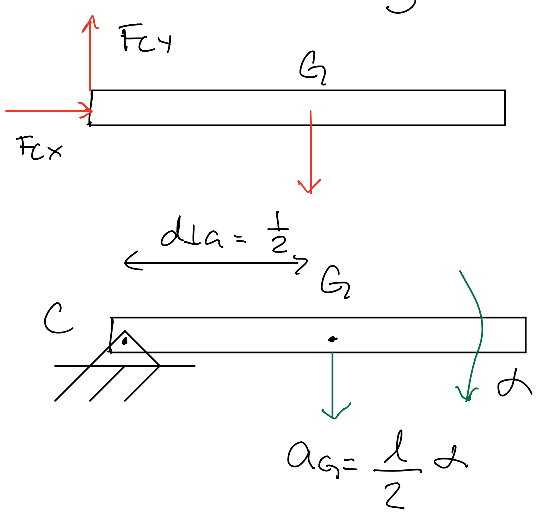
$$|\bar{r}_{AG} \times m\bar{a}_G| = r_{AG}\, m a_G \sin\theta = \begin{cases} m a_G\, d_{\perp a} \\ m\, r_{AG}\, a_{G\perp} \end{cases}$$

$$M_A = I_G \alpha + m a_G d_{\perp a}$$
$$M_A = I_G \alpha + m r_{AG} a_{G\perp}$$ (20)

Obs: M_A, α, $d_{\perp a}$, $a_{G\perp}$ tas med tecken.

Vektoriellt för svårare problem.

Ex: Fallande stång (alt. lösning).



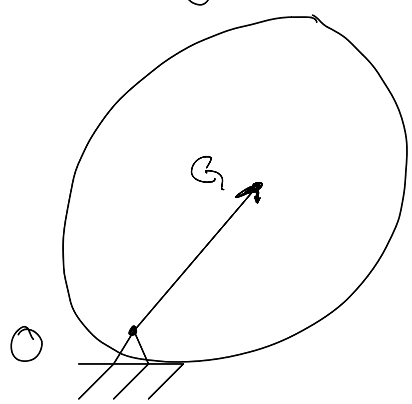
Euler II:

$$M_C = I_G \alpha + m a_G d_{\perp a}$$

$$\overset{\curvearrowleft}{C}: -mg\frac{l}{2} = -\frac{ml^2}{12}\alpha - m\frac{l}{2}\alpha\left(\frac{l}{2}\right)$$

$$\Leftrightarrow \alpha = \frac{3g}{2l}, \curvearrowright$$ ■

Momentlagen, punkt O fix i i-ram och kroppe



$\bar{r}_G := \bar{r}_{OG}$

O fix i i-ram $\Rightarrow \bar{M}_O = \dot{\bar{h}}_O$

$$\boxed{\bar{h}_O = I_O \bar{\omega}} \quad (21)$$

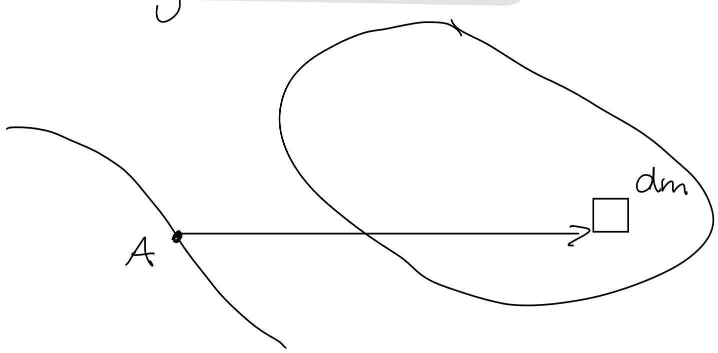
$I_O = I_G + m r_G^2$, kroppens masströghetsmoment map O, se nedan.

↑ konstant

$$\therefore \boxed{\bar{M}_O = I_O \bar{\alpha}} \quad (22)$$

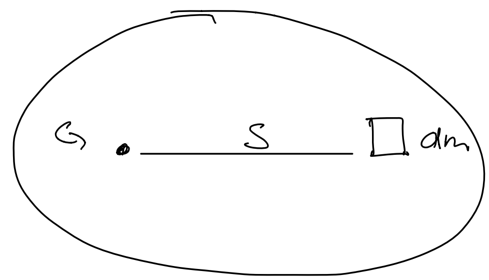
O fix i i-ram och kropp.

Masströghetsmoment, tunna skivor



Def: Masströghetsmomentet map godtyckligt A,

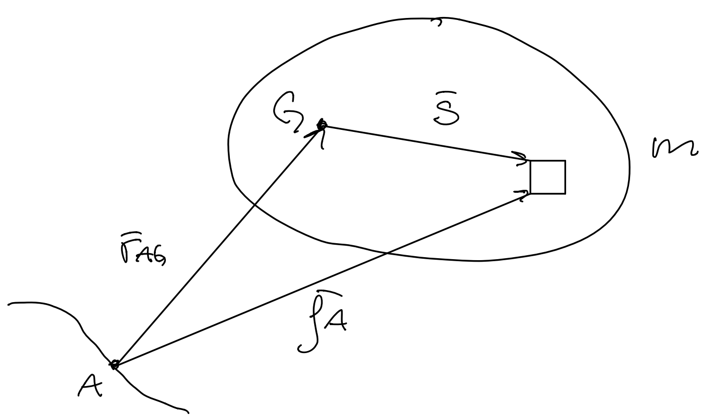
$$I_A = \int \rho_A^2 \, dm$$



$$I_G = \int s^2 \, dm$$

Huygens SATS:

$$I_A = I_G + m r_{AG}^2$$

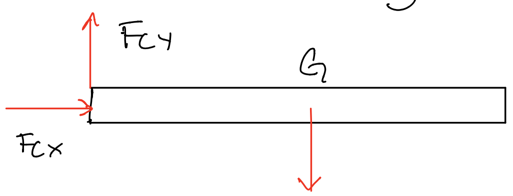


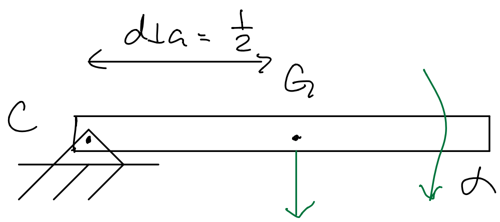
$$I_A = \int |\rho_A|^2 dm = \int (\bar{r}_{AG} + \bar{s}) \cdot (\bar{r}_{AG} + \bar{s}) \, dm =$$

$$= \int (|\bar{r}_{AG}|^2 + |\bar{s}|^2 + 2\bar{r}_{AG} \cdot \bar{s}) \, dm =$$

$$= \bar{r}_{AG}^2 \underbrace{\int dm}_{m} + I_G + 2\,\bar{r}_{AG} \cdot \underbrace{\int \bar{s}\, dm}_{\bar{0} \text{ enl. (10)}}$$ ▨

Ex: Fallande stång (alt. lösning).





$$a_G = \frac{l}{2}\alpha$$

Euler II:

$M_C = I_C \alpha$, C fix i i-ram och kropp.

$\curvearrowright C: \quad mg\frac{l}{2} = +I_C\alpha$

Huygens ⇒

$$I_C = \underbrace{I_G}_{\frac{ml^2}{12}} + m\left(\frac{l}{2}\right)^2 = \frac{ml^2}{3}$$

$\therefore \alpha = \frac{3g}{2l}, \curvearrowright$

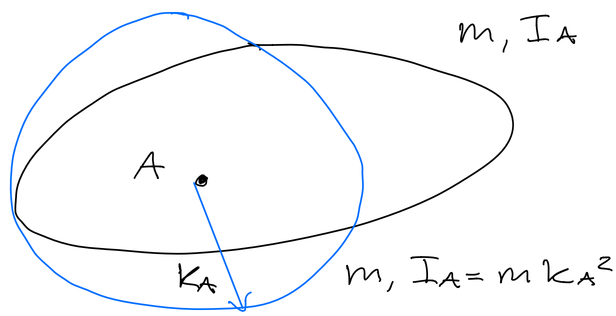
Def: Tröghetsradie

Tröghetsradie map godtycklig punkt A,

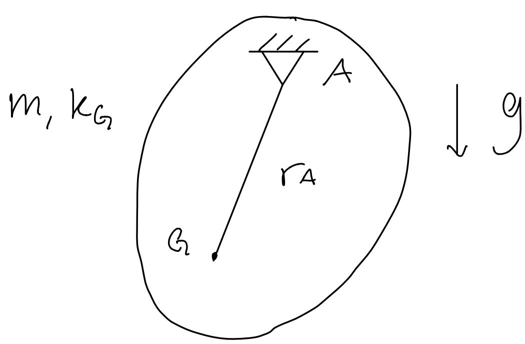
$$k_A = \sqrt{\frac{I_A}{m}},$$

så

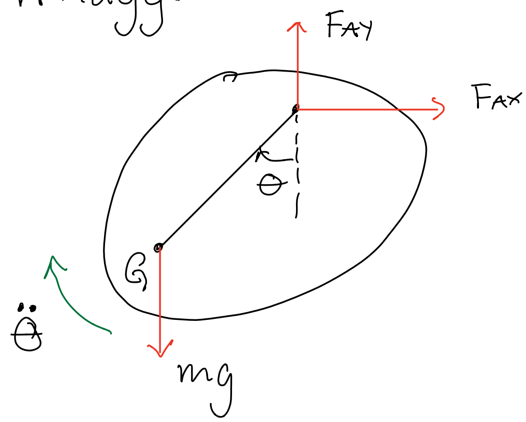
$$I_A = m k_A^2$$



Ex: Bestäm svängningstiden T för liten amplitud.



Frilägg:



Euler II:

$M_A = I_A \alpha$, A fix i i-ram och kropp.

$$\overset{\curvearrowright}{A}: \; -mg\, r_A \sin\theta = I_A \ddot{\theta} \iff$$

$$\ddot{\theta} + \frac{mg\, r_A}{I_A} \cdot \sin\theta = 0$$

$$I_A = [\text{Huygens}] = \underbrace{I_G}_{m k_G^2} + m r_A^2$$

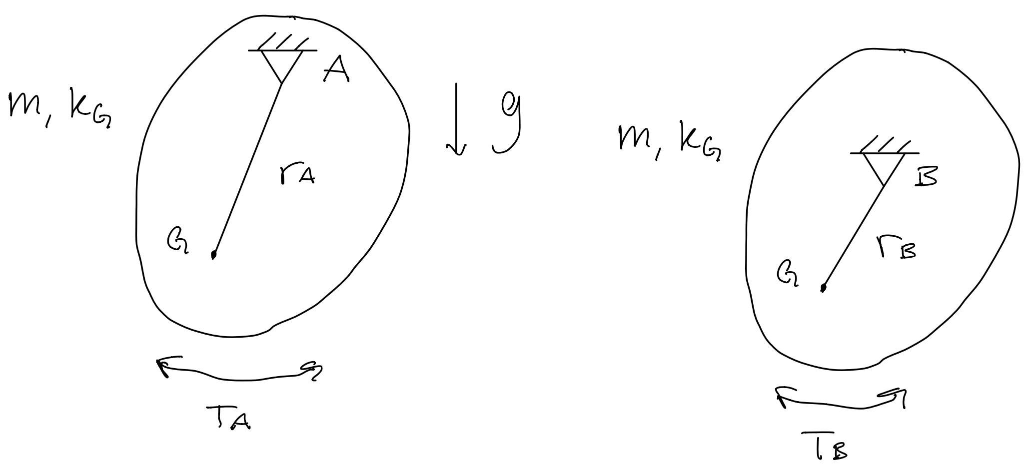
$$\therefore \quad \ddot{\theta} + \frac{g r_A}{k_G^2 + r_A^2} \cdot \sin\theta = 0 \qquad (23)$$

θ liten $\Rightarrow$

$$\ddot{\theta} + \underbrace{\frac{g r_A}{k_G^2 + r_A^2}}_{\omega_n^2} \cdot \theta = 0, \qquad \omega_n = 2\pi f = \frac{2\pi}{T}$$

$$\Rightarrow T = 2\pi \sqrt{\frac{k_G^2 + r_A^2}{g r_A}} \qquad (24)$$

Ex: Hitta B så att perioderna $T_A = T_B$.



(23) $\Rightarrow$

$$\begin{cases} \ddot{\theta} + \dfrac{g r_A}{k_G^2 + r_A^2} \sin\theta = 0 \\ \ddot{\theta} + \dfrac{g r_B}{k_G^2 + r_B^2} \sin\theta = 0 \end{cases}$$

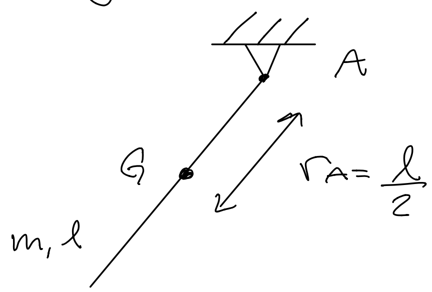
Samma period $\Rightarrow$

$$\frac{r_A}{k_G^2 + r_A^2} = \frac{r_B}{k_G^2 + r_B^2} \Leftrightarrow$$

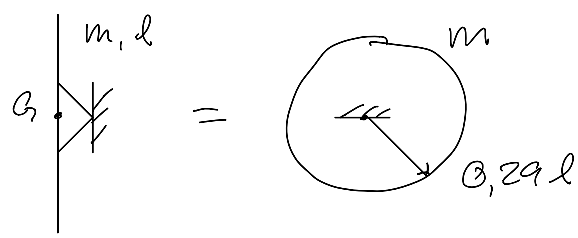
$$\Leftrightarrow r_A r_B^2 + r_A k_G^2 - r_B(k_G^2 + r_A^2) = 0 \Leftrightarrow$$

$$\Leftrightarrow r_B = \frac{k_G^2}{r_A} \quad (25) \quad \text{eller} \quad r_B = r_A$$

Stång:

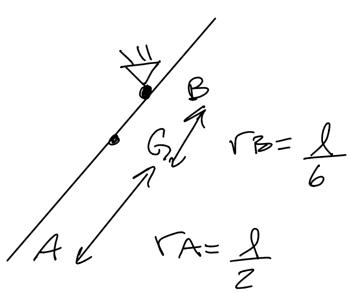


$$I_G = \frac{ml^2}{12} \Rightarrow k_G^2 = \frac{I_G}{m} = \frac{l^2}{12}, \quad k_G = 0{,}29\,l.$$



(25) $\Rightarrow$

$$r_B = \frac{l^2}{12} \Big/ \frac{l}{2} = \frac{l}{6}.$$



Svängningstiden, ~~θ~~ liten

$$T \overset{(24)}{=} 2\pi\sqrt{\frac{k_G^2 + r_A^2}{g r_A}} \overset{(25)}{=} 2\pi\sqrt{\frac{r_A r_B + r_A^2}{g r_A}} =$$

$$= 2\pi\sqrt{\frac{r_B + r_A}{g}} = 2\pi\sqrt{\frac{r_{AB}}{g}}.$$