Föreläsning 8

TMME04 – Mekanik II

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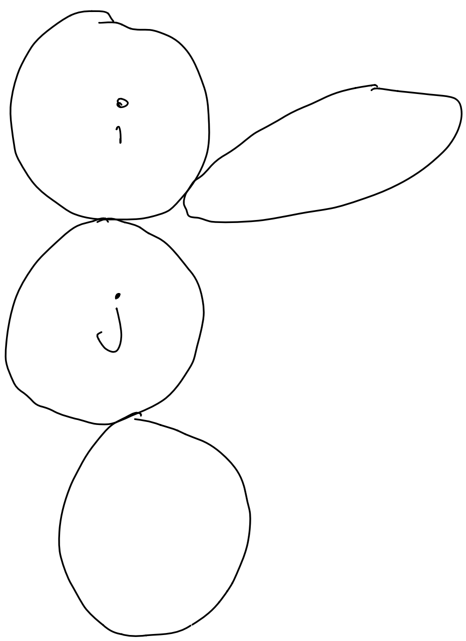
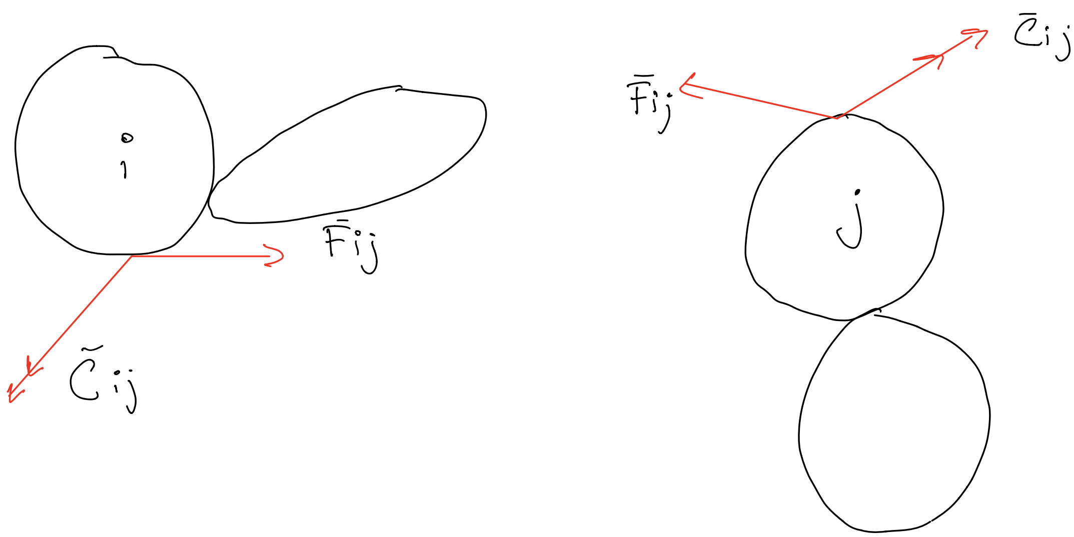


Frilägg kropp i från kropp j:

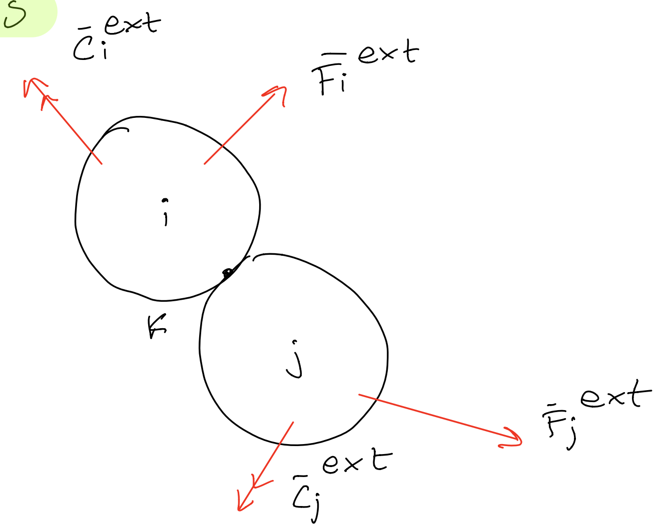


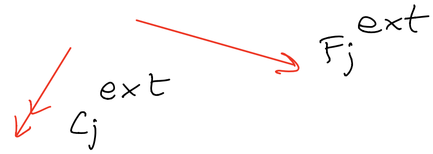
SATS:

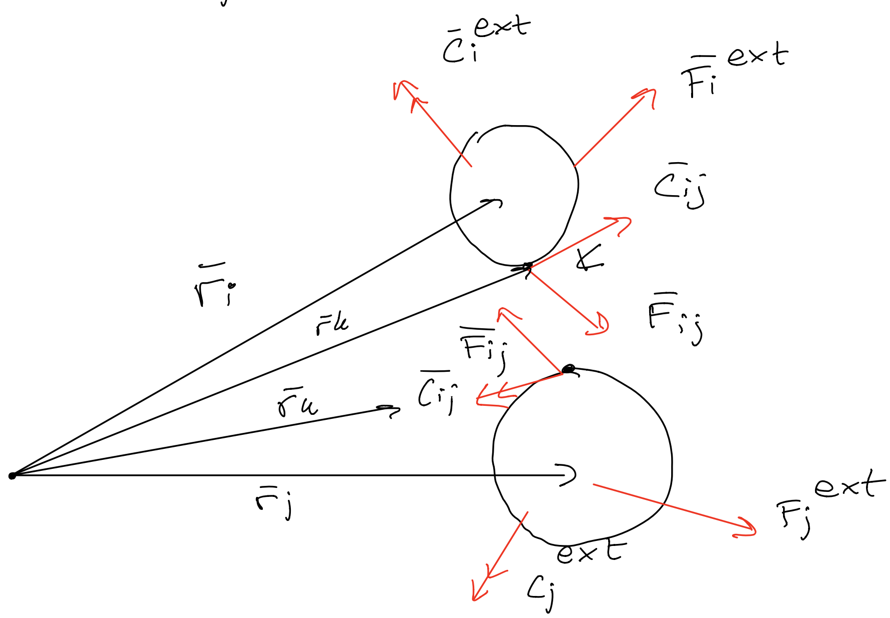
Lagen om verkan och motverkan, Newton III.

$$\bar{F}_{ij} = -\bar{F}_{ji}$$
$$\bar{C}_{ij} = -\bar{C}_{ji}$$

Bevis







Euler I for

$$i: \bar{F}_i^{ext} + F_{ij} = \dot{\bar{p}}^i \quad (1)$$

$$j: \bar{F}_j^{ext} + F_{ji} = \dot{\bar{p}}^j \quad (2)$$

$$i+j: \bar{F}_i^{ext} + \bar{F}_j^{ext} = \dot{\bar{p}}^i + \dot{\bar{p}}^j \quad (3)$$

(1) + (2) - (3):

$$\bar{F}_{ij} + F_{ji} = 0 \iff F_{ij} = -F_{ji} \quad \blacksquare$$

Omskrivningar av Euler I och II

Euler I: Kraftlagen

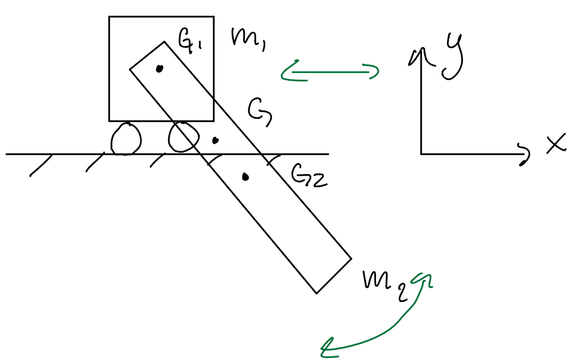
$$\bar{F}^{ext} = m\bar{a}_G,$$

$\bar{F}^{ext}$ extern kraft på systemet.

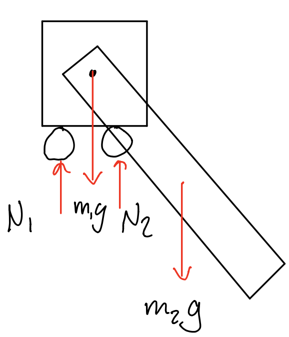
m systemets totala massa

G systemets masscentrum

Ex:



Frilägg:

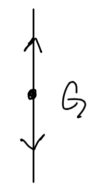


Ingen friktion så ingen extern kraft i $\hat{x}$.

$$\bar{F}_x^{ext} = 0 \Rightarrow a_{Gx} = 0 \Rightarrow V_{Gx} \text{ konstant.}$$

Startar i vila ger

$$V_{Gx} = 0 \quad \forall t.$$



$$\boxed{\bar{F}^{ext} = \sum m_i a_{Gi}}$$

$$r_G = \frac{m_1 \bar{r}_{G1} + m_2 r_{G2}}{m_1 + m_2}$$

Euler II: Momentlagen

$$\boxed{\bar{M}_A^{ext} = \sum I_{Gi} \bar{\alpha}_i + \sum r_{AGi} \times m_i a_{Gi}, \quad A \text{ godtycklig}}$$

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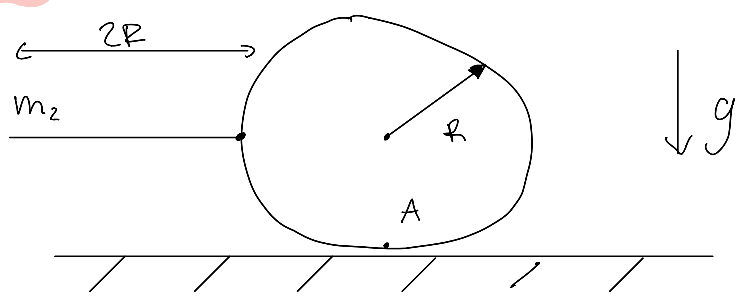
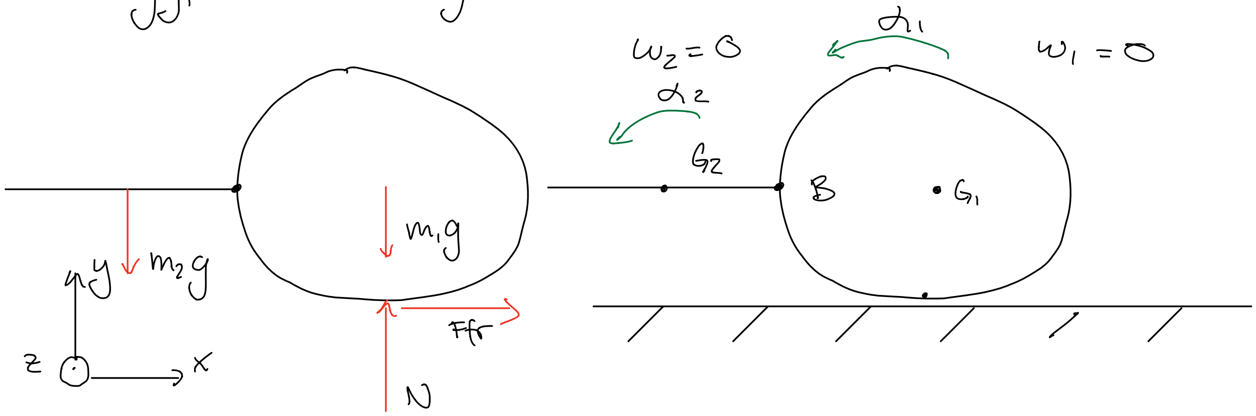


Givet: Släpps från vila då $t = 0$

Sökt: α_1, α_2 vid $t = 0$

Frilägg, skiva + stång vid $t=0$



Euler II

$$\bar{M}_A^{ext} = \sum_{i=1}^{2} I_{G_i}\bar{\alpha}_i + \sum_{j=1}^{2} \bar{r}_{AG_i} \times m_i \bar{a}_{G_i}$$

$$\bar{M}_A^{ext}: m_2 g \cdot 2R\,\hat{z} = \underbrace{I_{G_1}}_{\frac{m_1R^2}{2}}\alpha_1\hat{z} + \underbrace{I_{G_2}}_{\frac{m_2(2R)^2}{12}=\frac{m_2R^2}{3}}\alpha_2\hat{z} + R\hat{y}\times m_1(-R\alpha_1\hat{x}) +$$

$$+ (R\hat{y} - 2R\hat{x}) \times m_2\bar{a}_{G_2} \quad (1)$$

Kinematik

Skivan

$$\bar{a}_B = \bar{a}_{G_1} + \alpha_1 \times r_{GB} - \underbrace{\omega_1^2}_{=0} r_{GB} = -R\alpha\hat{x} + \alpha_1\hat{z}\times(-R\hat{x}) =$$

$$= -R\alpha_1\hat{x} - R\alpha_1\hat{y}$$

Stången

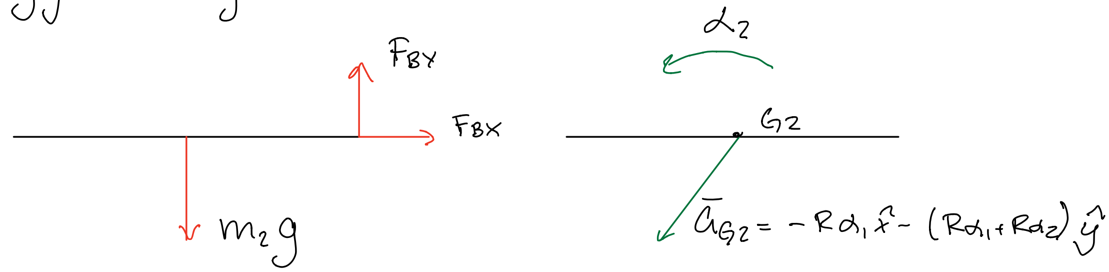
$$a_{G_2} = \bar{a}_B + \bar{\alpha}_2 \times \bar{r}_{BG_2} - \underbrace{\omega_2^2}_{=0}\bar{r}_{BG_2} = -R\alpha_1\hat{x} - R\alpha_1\hat{y} +$$

$$+ \alpha_2\hat{z}\times(-R\hat{x}) = -R\alpha_1\hat{x} - (R\alpha_1 + R\alpha_2)\hat{y}$$

Insättning i (1) $\Rightarrow \dots \Rightarrow$

$$2m_2 g R = \frac{3}{2} m_1 R^2 \alpha_1 + 3m_2 R^2 \alpha_1 + \frac{7}{3} m_2 R^2 \alpha_2 \quad (2)$$

Frilägg stången vid $t=0$



Euler II

$$\bar{M}_B = I_{G2}\bar{\alpha}_2 + \bar{r}_{BG2} \times m_2 a_{G2}$$

$$\bar{M}_B : m_2 g R \hat{z} = \underbrace{I_{G2}}_{\frac{m_2 R^2}{3}} \alpha_2 \hat{z} +$$

$$\underbrace{+(-R\hat{x}) \times m_2 \left(-R\alpha_1 \hat{x} - (R\alpha_1 + R\alpha_2)\hat{y}\right)}_{m_2 R^2 (\alpha_1 + \alpha_2)\hat{z}} \Leftrightarrow$$

$$\Leftrightarrow m_2 g R = \frac{m_2 R^2}{3} \cdot \alpha_2 + m_2 R^2 (\alpha_1 + \alpha_2) \quad (3)$$

(2), (3) ger:

$$\alpha_1 = \frac{m_2 g}{R(6m_1 + 5m_2)}, \circlearrowleft \qquad \alpha_2 = \frac{\frac{9}{2} m_1 g + 3_2 g}{R(6m_1 + 5m_2)}, \circlearrowleft$$

$$\bar{F} = m\bar{a} : m = 0 \Rightarrow \bar{F} = 0$$

$|F_{fr}| \le \mu_s N$

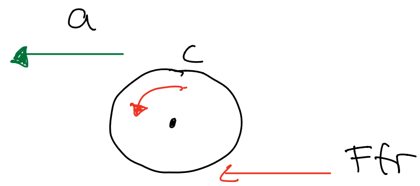
$\mu_{s,min}$ så ej börjar glida

Euler I

$$\bar{F}^{ext} = \sum_{i=1}^{2} m_i a_{G_i}$$

$$\hat{x}: \; F_{fr} = m_1 a_{G_1 x} + m_2 a_{G_2 x} = m_1(-R\alpha_1) + m_2(-R\alpha_1) =$$

$$= -(m_1+m_2)R\alpha_1 = -\frac{(m_1+m_2)\, m_2 g}{6m_1+5m_2}$$



$$\hat{y}: \; N - (m_1+m_2)g = m_1 a_{G_1 y} + m_2 a_{G_2 y} =$$

$$= 0 - m_2 R(\alpha_1+\alpha_2)$$

$$\Rightarrow N = \frac{6m_1^2 + \frac{13}{2} m_1 m_2 + m_2^2}{6m_1+5m_2}\, g$$

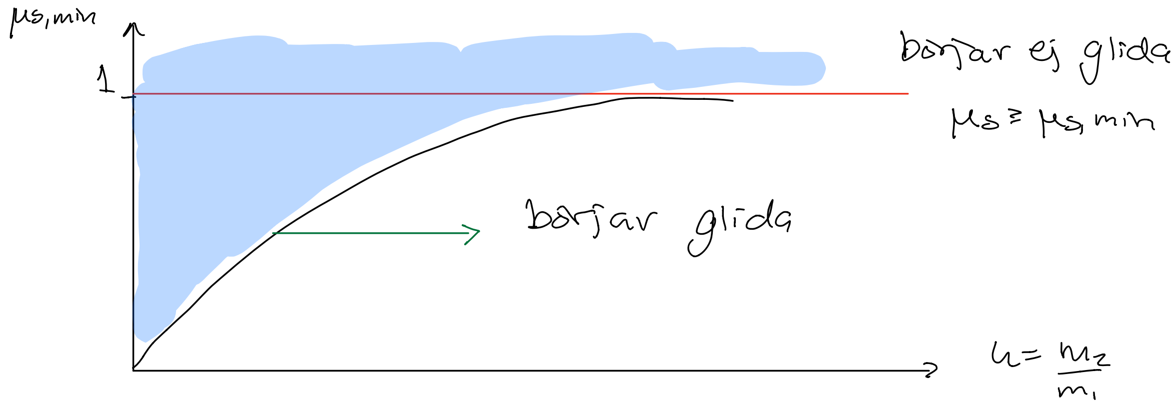
Börjar ej glida om

$|F_{fr}| \le \mu_s N.$

$$\therefore \; \mu_{s,min} = \frac{|F_{fr}|}{N} = \frac{m_2(m_1+m_2)}{6m_1^2 + m_2^2 + \frac{13}{2} m_1 m_2}$$

$m_2 = k m_1 \Rightarrow$

$$\mu_{s,min} = \frac{k(1+k)}{6+k^2+\frac{13}{2}k}$$



$\mu_{s,min}$
1
börjar ej glida
$\mu_s \geq \mu_{s,min}$
börjar glida
$u = \frac{m_2}{m_1}$