

# Föreläsning 1

TMME12 – Mekanik I

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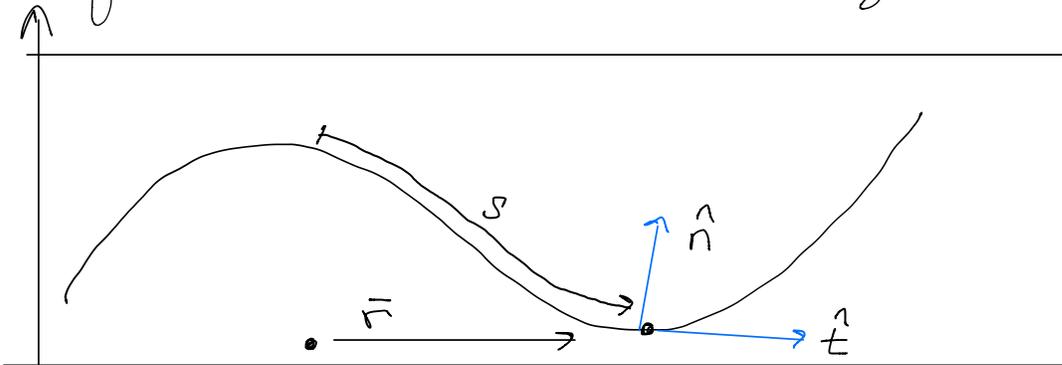
<https://www.instagram.com/olwettergren/>

$$\hat{t} \cdot \hat{t} = 1 \Rightarrow \frac{d\hat{t}}{ds} \cdot \hat{t} + \hat{t} \cdot \frac{d\hat{t}}{ds} = 0$$

$$\Rightarrow \frac{d\hat{t}}{ds} \cdot \hat{t} = 0$$

$$\therefore \frac{d\hat{t}}{ds} \perp \hat{t}$$

Hastighet och acceleration: naturliga basen (tn)



Mentalram, dvs fix Referensram.

s-båglängd,  $\hat{n}$ -normalen

• Hastighet:

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d\vec{r}}{ds} \cdot \frac{ds}{dt} = \hat{t} \frac{ds}{dt}$$

$$\hat{t} = \frac{d\vec{r}}{ds}$$

$$\hat{n} = \frac{d\hat{t}}{ds} / \left| \frac{d\hat{t}}{ds} \right|$$

$$\rho = \frac{1}{\kappa}$$

$$\vec{v} = \dot{s} \hat{t}$$

• Fart:

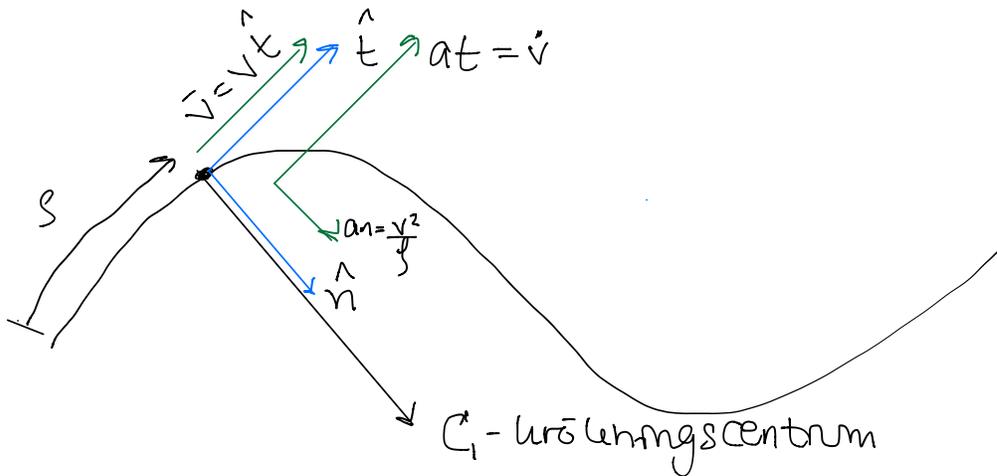
$$|\vec{v}| = |\dot{s}| = |v|$$

• Acceleration

$$\begin{aligned} \vec{a} = \dot{\vec{v}} &= \dot{v} \hat{t} + v \dot{\hat{t}} = \dot{v} \hat{t} + v \underbrace{\frac{d\hat{t}}{ds}}_{(2) \frac{1}{\rho} \hat{n}} \underbrace{\frac{ds}{dt}}_{j=v} \\ &\stackrel{(4)}{=} \dot{v} \hat{t} + \frac{v^2}{\rho} \hat{n} \end{aligned}$$

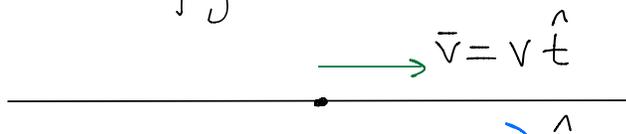
$$\vec{a} = \dot{v} \hat{t} + \frac{v^2}{\rho} \hat{n}$$

} krökningsradie.



$a_t = \dot{v}$ : fartändringsterm  
 $a_n = \frac{v^2}{\rho}$ : riktningssändringsterm, centripetal accelerat.

Ex: Rätlinjig rörelse



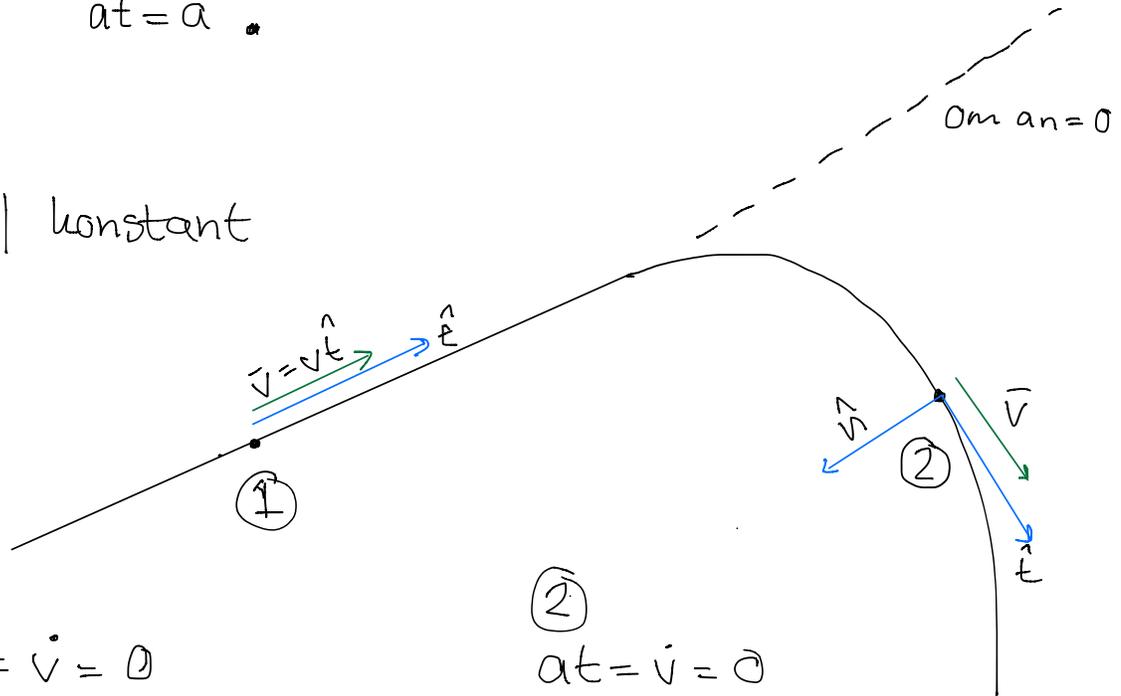


$$f \rightarrow \infty \Rightarrow a_n = 0$$

$$\therefore \bar{a} = \frac{d\hat{v}}{dt} \hat{t}$$

Ex.

$|\bar{v}|$  konstant



①

$$a_t = \dot{v} = 0$$

$$a_n = \frac{v^2}{f} = 0 \quad \text{ty } f \rightarrow \infty$$

$$\therefore \bar{a} = \bar{0}$$

②

$$a_t = \dot{v} = 0$$

$$a_n = \frac{v^2}{f} \neq 0$$

$$\therefore \bar{a} \neq \bar{0}$$

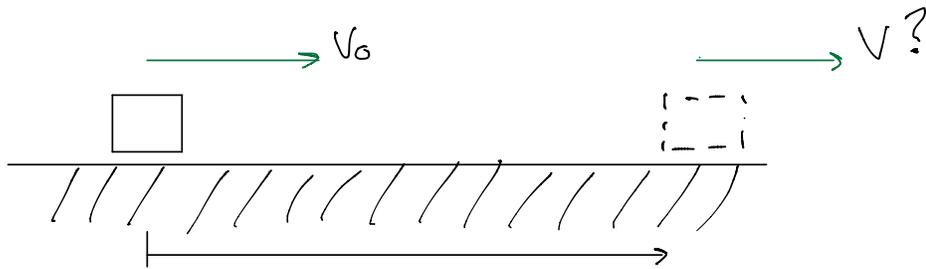
OBS: Krövt bana  $\Rightarrow \bar{a} \neq \bar{0}$ !

Varignons Sats

$$\left. \begin{aligned} v &= \frac{ds}{dt} \\ a_t &= \dot{v} = \frac{dv}{dt} \end{aligned} \right\} \Rightarrow \begin{aligned} dt &= \frac{ds}{v} \\ dt &= \frac{dv}{a_t} \end{aligned} \Rightarrow$$

$$a_t ds = v dv$$

Ex:



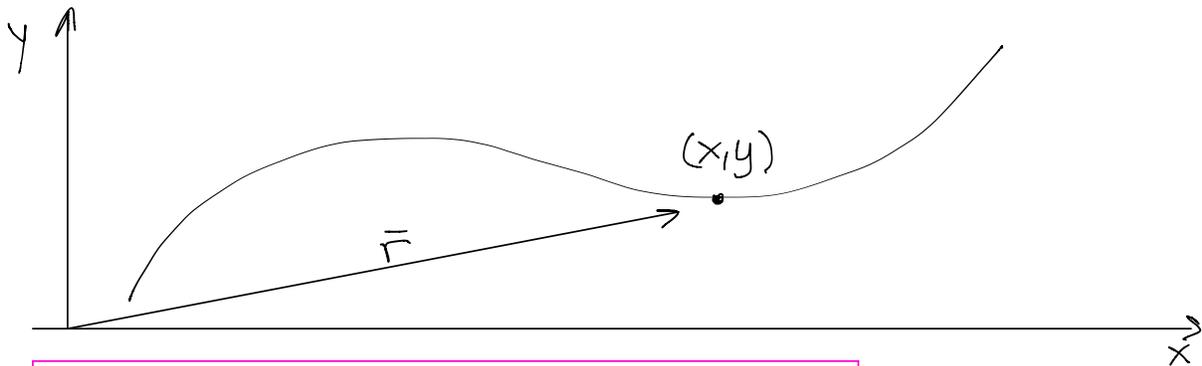
$a t = a \text{ constant}$

$$\Rightarrow a ds = v dv \Rightarrow \int_0^s a ds = \int_{v_0}^v v dv \Leftrightarrow$$

$\begin{matrix} \text{slot} \\ \nearrow s \\ \int_0^s \\ \leftarrow \text{start} \end{matrix}$

$$\Leftrightarrow as = \frac{1}{2} (v^2 - v_0^2) \Rightarrow v = \sqrt{v_0^2 + 2as} .$$

Kartesischer Koordinaten



$$\vec{r} = x \hat{x} + y \hat{y}$$

$$\vec{v} = \dot{x} \hat{x} + \dot{y} \hat{y}$$

(fixa)

$$\vec{a} = \ddot{x} \hat{x} + \ddot{y} \hat{y}$$

Variation

$$\dot{x} = \frac{dx}{dt}, \quad \ddot{x} = \frac{d\dot{x}}{dt} \Rightarrow$$

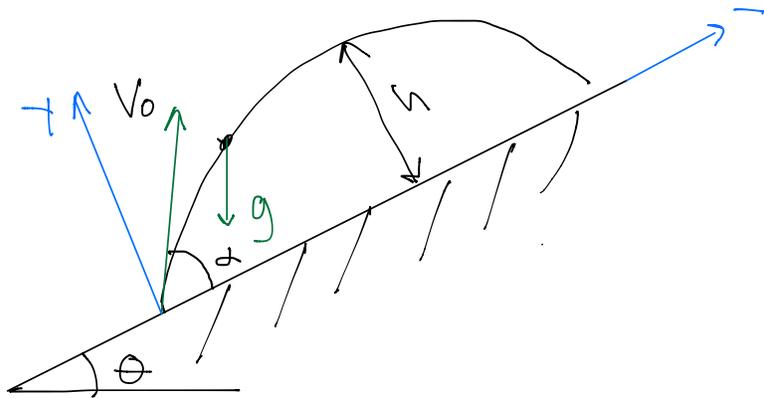
Varignon  
at  $ds = v dv$

$$\begin{aligned} \ddot{x} dx &= \dot{x} d\dot{x} \\ \ddot{y} dy &= \dot{y} d\dot{y} \end{aligned} \quad (6)$$

Ex Söker maximala höjden över plan

Givet:

$$\bar{a} = g, \downarrow$$

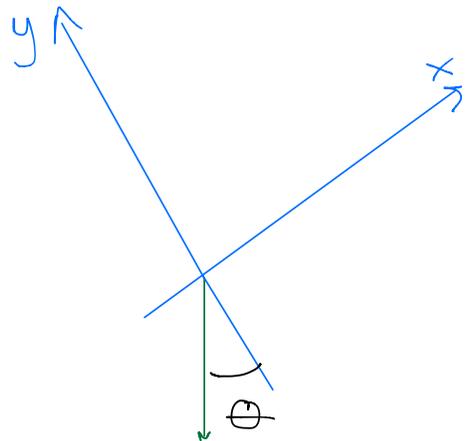


$$h = y_{\max}$$

$$(6) \Rightarrow \underbrace{\ddot{y}}_{-g \cos \theta} dy = \dot{y} d\dot{y} \Rightarrow$$

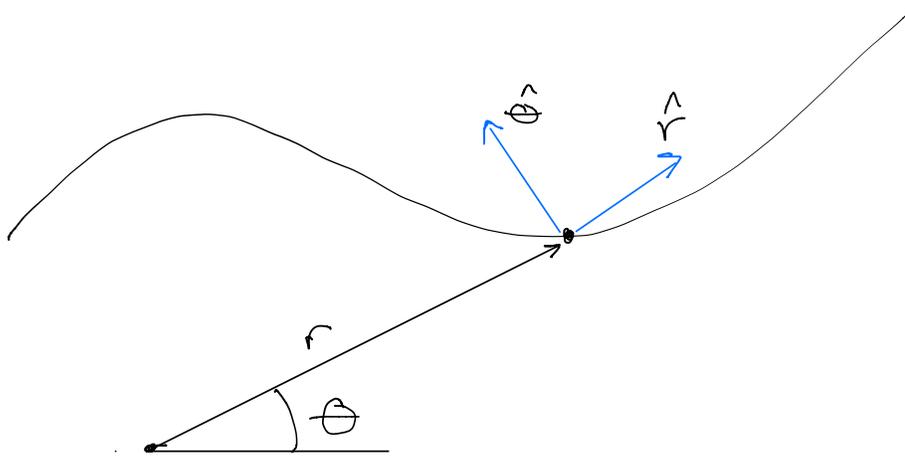
$$\Rightarrow \int_0^h \underbrace{\ddot{y}}_{-g \cos \theta} dy = \int_0^0 \dot{y} d\dot{y} \Rightarrow$$

$$v_0 \cdot \sin \alpha$$



$$\Rightarrow -g \cos \theta \cdot h = \left[ \frac{\dot{y}^2}{2} \right]_0^d \cdot v_0 \cdot \sin \alpha \Rightarrow h = \frac{v_0^2 \sin^2 \alpha}{2g \sin \alpha}$$

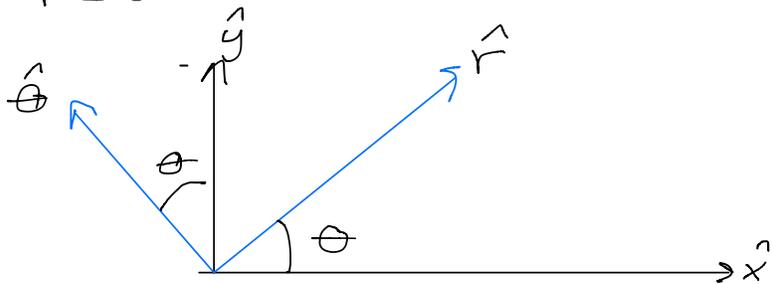
## Polare Koordinaten



(fix linje)

$$\vec{r} = r \hat{r}$$

$$\vec{v} = \dot{r} \hat{r} + r \dot{\hat{r}}, \quad \dot{\hat{r}} = ?$$



$$\hat{r} = \cos \theta \hat{x} + \sin \theta \hat{y}$$

$$\hat{\theta} = -\sin \theta \hat{x} + \cos \theta \hat{y}$$

$$\therefore \dot{\hat{r}} = -\sin \theta \cdot \dot{\theta} \hat{x} + \cos \theta \dot{\theta} \hat{y} = \dot{\theta} \hat{\theta}$$

$$\dot{\hat{\theta}} = -\cos \theta \cdot \dot{\theta} \hat{x} - \sin \theta \dot{\theta} \hat{y} = -\dot{\theta} \hat{r}$$

$$\vec{v} = \dot{r} \hat{r} + r \dot{\theta} \hat{\theta}$$

$\dot{\theta}$  i rad/s

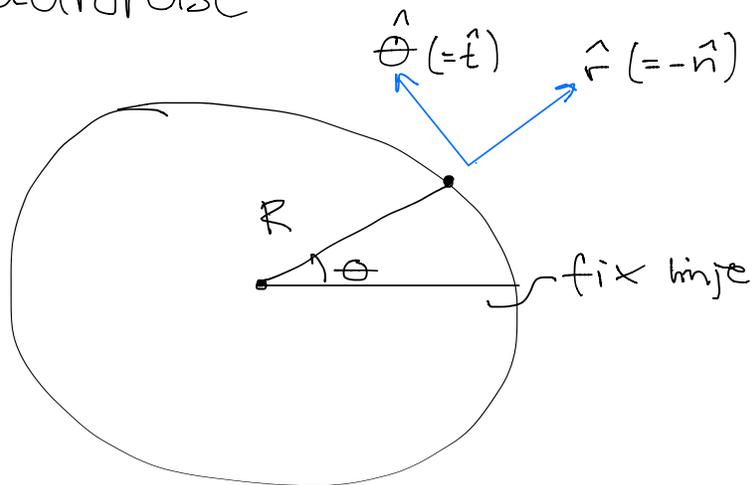
$$\begin{aligned}\vec{a} = \dot{\vec{v}} &= \ddot{r}\hat{r} + \dot{r}\dot{\hat{r}} + \dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} + r\dot{\theta}\dot{\hat{\theta}} \\ &= \ddot{r}\hat{r} + 2\dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} - r\dot{\theta}^2\hat{r}\end{aligned}$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$$

(b)  $\Rightarrow$

$$\begin{aligned}\dot{r} dr &= r dr \\ \dot{\theta} d\theta &= \dot{\theta} d\theta\end{aligned}$$

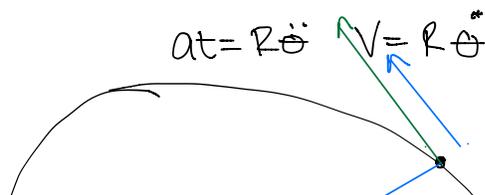
Ex: Cirkelrörelse



$r = R$ , konstant

$$\begin{aligned}\vec{v} &= R\dot{\theta}\hat{\theta} \\ \vec{a} &= -R\dot{\theta}^2\hat{r} + R\ddot{\theta}\hat{\theta}\end{aligned}$$

Allt; naturliga basen.



$$\vec{v} = R\dot{\theta}\hat{e}$$

$$\vec{a} = R\ddot{\theta}\hat{e} + R\dot{\theta}^2\hat{n}$$

