Lektion 10

TAMS24 – Statistisk teori

Skriven av Oliver Wettergren
oliwe188@student.liu.se
https://www.instagram.com/olwettergren/

Lektionsgenomgång

$$Y = \beta_0 + \beta_1 x_1 + \dots + \beta_k x_k + \varepsilon, \quad \varepsilon \sim N(0, \sigma)$$

$$\Rightarrow \{(x_{i1}, \dots, x_{ik}), y_i\}, \quad i = 1, 2, \dots, n$$

$$\Rightarrow \hat{\beta} = \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \vdots \\ \hat{\beta}_k \end{pmatrix} = (\mathbb{X}^T \mathbb{X})^{-1} \mathbb{X}^T y,$$

Where

$$\mathbb{X} = \begin{pmatrix} 1 & x_{11} & \dots & x_{1k} \\ 1 & x_{21} & \dots & x_{2k} \\ \vdots & & \vdots & \\ 1 & x_{n1} & \dots & x_{nk} \end{pmatrix}, \quad y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

Given

$$\mathbb{x} = \begin{pmatrix} 1 \\ x_1 \\ \vdots \\ x_k \end{pmatrix} \rightarrow Y \Rightarrow$$

$$\text{P.I} \quad I_Y = \mathbb{x}^T \hat{\beta} \mp t_{\alpha/2}(n-k-1) \cdot s \cdot \sqrt{\mathbb{x}^T (\mathbb{X}^T \mathbb{X})^{-1} \mathbb{x} + 1}$$

$$\mu = E(Y) \Rightarrow \text{C.I} \quad I_\mu = \mathbb{x}^T \hat{\beta} \mp t_{\alpha/2}(n-k-1) \cdot s \sqrt{\mathbb{x}^T (\mathbb{X}^T \mathbb{X})^{-1} \mathbb{x} + 1}$$

Model 1.

$$Y = \beta_0 + \beta_1 x_1 + \beta_k x_k + \varepsilon$$

Model 2:

$$Y = \beta_0 + \beta_1 x_1 + \dots + \beta_k x_k + \beta_{k+1} x_{k+1} + \dots + \beta_{k+p} x_{k+p} + \varepsilon,$$

$$\varepsilon \sim N(0, \sigma)$$

$$\begin{cases} H_0: \beta_{k+1} = \dots = \beta_{k+p} = 0 \\ H_1 \text{ at least one } \beta_{k+j} \neq 0 \quad j=1,2,\dots p. \end{cases}$$

fact:

$$\frac{(SSE^{(1)} - SSE^{(2)})/p}{SSE^{(2)}/(n-k-p-1)} \sim F(p, n-k-p-1).$$

G 2.7

CI $\alpha = 5\%$, $x_1 = 50$
$x_2 = 18$

$$\mathbb{X} = \begin{pmatrix} 1 \\ 50 \\ 18 \end{pmatrix} \quad \mathbb{X}^T = (1 \ 50 \ 18)$$

$\mathbb{X}^T(\mathbb{X}^T\mathbb{X})^{-1}\mathbb{X}$

$$(1 \ 50 \ 18)\begin{pmatrix} 2.51724 & & \\ -0.01926 & 0.00153 & \\ -0.06189 & -0.00347 & 0.01288 \end{pmatrix}\begin{pmatrix} 1 \\ 50 \\ 18 \end{pmatrix} = 0.11$$

$$I_\mu = \mathbb{X}^T\hat{\beta} \mp t_{\alpha/2}(n-k-1) \cdot s\sqrt{\mathbb{X}^T(\mathbb{X}^T\mathbb{X})^{-1}\mathbb{X}} = $$

$$\mathbb{X}^T\hat{\beta} = (1 \ 50 \ 18)\begin{pmatrix} -50.359 \\ 0.6712 \\ 1.2954 \end{pmatrix} = 6.5182$$

$t_{\alpha/2}(n-k-2) = t_{0.025}(18) = 2.11$ / =

$s^2 = \frac{SSE}{n-k-1} = \frac{188.8}{18} = 10.48$

$\Rightarrow 6.5182 - 2.10 \cdot \sqrt{10.48} \ \sqrt{0.11} = 4.2$

$\Rightarrow \quad + \quad = 8.8$ $\Rightarrow I_\mu(4.2, 8.8)$

Nej! Not happy ty

GR 2.8

$\alpha = 5\%, \quad x_1 = 15\,000 \quad x_2 = 287$

$$\mathbf{x} = \begin{pmatrix} 1 \\ \frac{15\,000}{60} \\ 287 \end{pmatrix} \Rightarrow \mathbf{x}^T = \begin{pmatrix} 1 & \frac{15\,000}{60} & \ln(287) \end{pmatrix}$$

$$\mathbf{x}^T(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{x} - 1 = \begin{pmatrix} 1 & \frac{15000}{60} & \ln(287) \end{pmatrix} \begin{pmatrix} 2.51724 & & \\ -0.01926 & 0.00153 & \\ -0.06189 & -0.00347 & 0.01288 \end{pmatrix} \cdot$$

$$\cdot \begin{pmatrix} 1 \\ \frac{15000}{60} \\ \ln(287) \end{pmatrix} - 1 = 1.161890$$

$$= \mathbf{x}^T \hat{\beta} = \begin{pmatrix} 1 & \frac{15000}{60} & \ln(287) \end{pmatrix} \begin{pmatrix} -782.7 \\ 0.6881 \\ 50.29 \end{pmatrix} = 173.94$$

$t_{\frac{\alpha}{2}}(14) = 2.14$

$$s^2 = \frac{SSE}{n-k-1} = \frac{6605.7}{14} = 471.6357$$

$$\therefore I_\Sigma = \mathbf{x}^T\hat{\beta} \pm t_{\frac{\alpha}{2}}(n-k-1) \cdot s\sqrt{\mathbf{x}^T(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{x} - 1} =$$

$$= 173.94 \pm 2.14 \cdot \sqrt{471}\sqrt{1.161890} = (123{,}7,\ 224.2)$$

JA! Bra shit.

GR. 2.9

a)