Föreläsning 10

TMME04 – Mekanik II

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SATS: Impulslagen

$$\boxed{\int_{t_1}^{t_2} \bar{F}\,dt = \bar{P}_2 - \bar{P}_1 \,, \quad \bar{p} = m\bar{v}_G,}$$

där rörelsemängden

$\bar{p}_1 = \bar{p}(t = t_1)$,
$\bar{p}_2 = \bar{p}(t = t_2)$

och

$\int \bar{F}dt$ kallas impulsen av $\bar{F}$

Bevis: Följer av $\bar{F} = \dot{\bar{p}}$

SATS: Impulsmomentlagen

$$\boxed{\int_{t_1}^{t_2} \bar{M}_P\,dt = (\bar{h}_P)_2 - (\bar{h}_P)_1 \,,}$$

P fix i i-ram.

$$\boxed{\int_{t_1}^{t_2} \bar{M}_G\,dt = (\bar{h}_G)_2 - (\bar{h}_G)_1}$$

$h_G = I_G \omega$

Bevis: Följer ur

$$\bar{M}_P = \dot{\bar{h}}_P$$

och

$$\bar{M}_G = \dot{\bar{h}}_G$$

Beräkning av $\bar{h}_A$,
A godtycklig punkt (plana problem)

A, B godtycklig ⇒

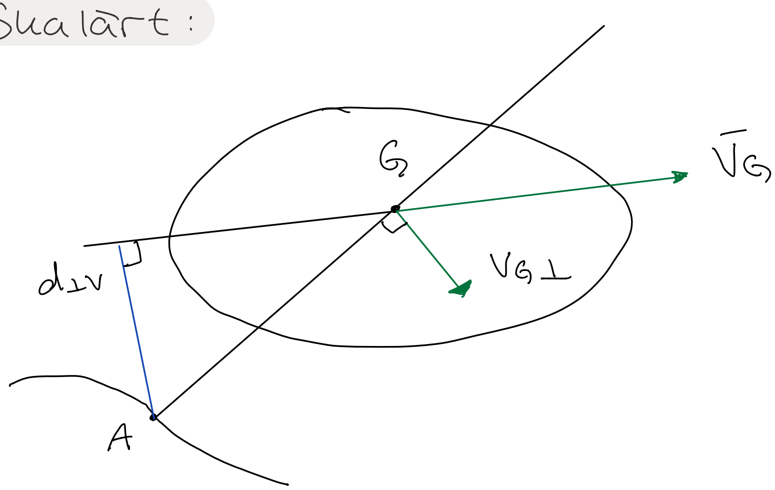
$$\bar{h}_A = \bar{h}_B + \bar{r}_{AB} \times m\bar{v}_G$$

Välj

$$B = G \Rightarrow$$

$$\boxed{\bar{h}_A = I_G\bar{\omega} + \bar{r}_{AG} \times m\bar{v}_G}$$

Skalärt:

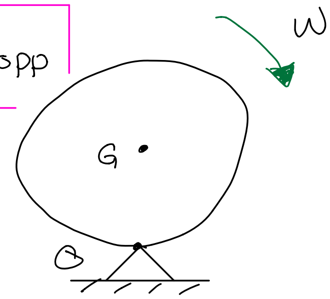


$$\boxed{\begin{aligned} h_A &= I_G\,\omega + m v_G\, d_{\perp v} \\ h_A &= I_G\,\omega + m r_{AG}\, v_{G\perp} \end{aligned}}$$

h_A, ω, $d_{\perp v}$, $v_{G\perp}$ tas med tecken

Specialfall:

$$\boxed{\bar{h}_O = I_O\,\bar{\omega}, \quad O \text{ fix i i-ram och kropp}}$$



OBS, I allmänhet:

$\bar{h}_P \neq I_P\bar{\omega}$, P fix i i-ram

$\bar{h}_C \neq I_C\bar{\omega}$, C kroppfix

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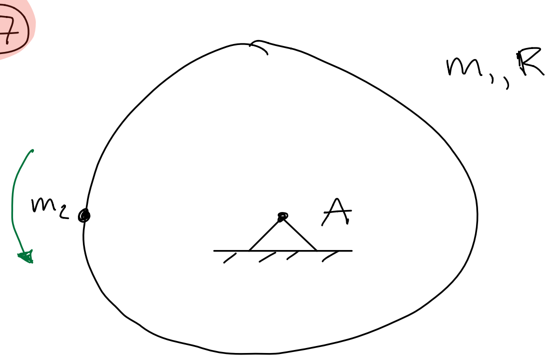
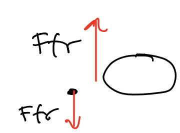


$\otimes g$



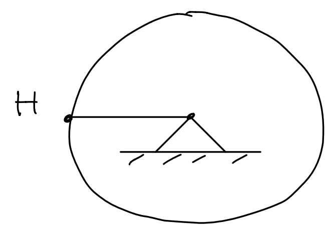
Givet:

Startar i vila, hamster spring ett varv moturs relativt skivan

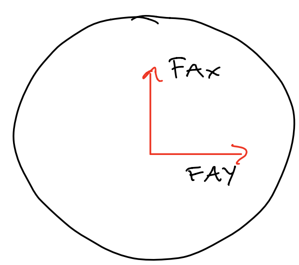
Sökt:

Vinkeländring för skivan

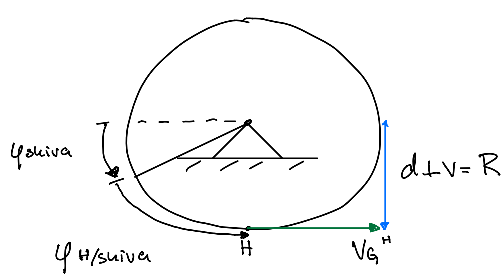
Start (1)



Frilägging av m_1+m_2, godtyckligt läge



Godtyckligt läge (2)



Impulsmomentlagen

$$\int M_A^{ext} dt = \sum_{i=1}^{2} (h_A^i)_2 - \sum_{i=1}^{2} (h_A^i)_1 ,$$

A fix i i-ram.

$\curvearrowleft A: \quad 0 = \left(h_A^{skiva}\right)_z + \left(h_A^{H}\right)_z - 0$ ty vila

$$\left(h_A^{skiva}\right)_z = I_A^{skiva}\,\dot{\varphi}_{skiva} = \frac{1}{2} m_1 R^2 \dot{\varphi}_{skiva},$$

A även fix i skivan

$$\left(h_A^{H}\right)_z = \underbrace{I_G^{H}}_{=0} \omega^{H} + m_2 \underbrace{v_G^{H}}_{R(\dot{\varphi}_{skiva} + \dot{\varphi}_{H/skiva})} \underbrace{d_{\perp v}}_{R}$$

cirkelrörelse

$$\therefore \quad \frac{1}{2} m_1 R^2 \dot{\varphi}_{skiva} + m_2 R^2 \left(\dot{\varphi}_{skiva} + \dot{\varphi}_{H/skiva}\right) = 0$$

$$\dot{\varphi}_{skiva} = \frac{d\varphi_{skiva}}{dt}$$

Integrera $\Rightarrow$

$$\left(\frac{1}{2} m_1 + m_2\right) R^2 \int_0^{\varphi_{skiva}} d\varphi_{skiva} + m_2 R^2 \int_0^{2\pi} d\varphi_{H/skiva} = 0$$

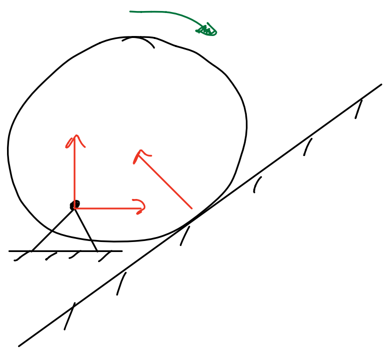
$$\Leftrightarrow \varphi_{skiva} = \frac{-2\pi m_2}{\frac{1}{2} m_1 + m_2}, \quad \curvearrowleft$$

Stöt

Vi kan inte använda Eulers lagar

Stötimpulslagen

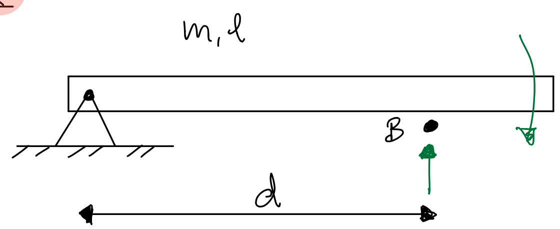
$$L^{s,ext} = \sum_{i=1}^{n_k} p^{i,e} - \sum_{i=1}^{n_u} p^{i,f}$$



Stötimpulsmomentlagen

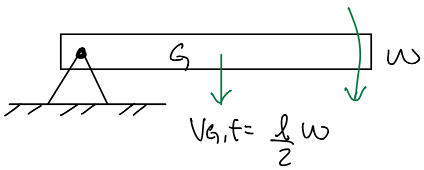
$$A_p^{s,ext} = \sum_{i=1}^{n_k} h_p^{i,e} - \sum_{i=1}^{n_u} h_p^{i,f}$$

Ex:

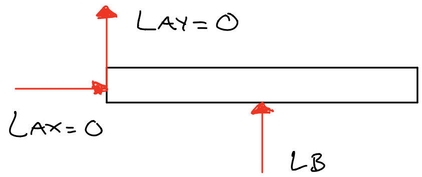


Bestäm d så att ingen stötimpuls fås vid A.

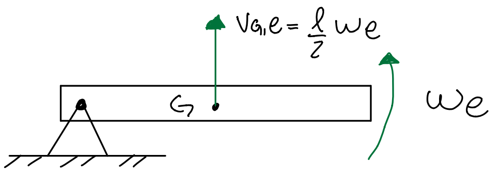
Precis före stöten:



Vid stöten:



Precis efter stöten:



Stötimpulslagen

$$L^s = \bar{p}_e - \bar{p}_f \,, \quad \bar{p} = m\bar{v}_G$$

$\longrightarrow$: $L_{Ax} = 0$, oberoende av d

$$\uparrow: \underbrace{L_{Ay}}_{=0} + L_B = m\frac{\ell}{2}\left(\omega_e - (-\omega)\right) \quad (1)$$

Stötimpulsmomentlagen

$A_A^s = h_{A,e} - h_{A,f}$, A fix i i-ram

$h_A = I_A\omega$, A även fix i stången.

hävarmen

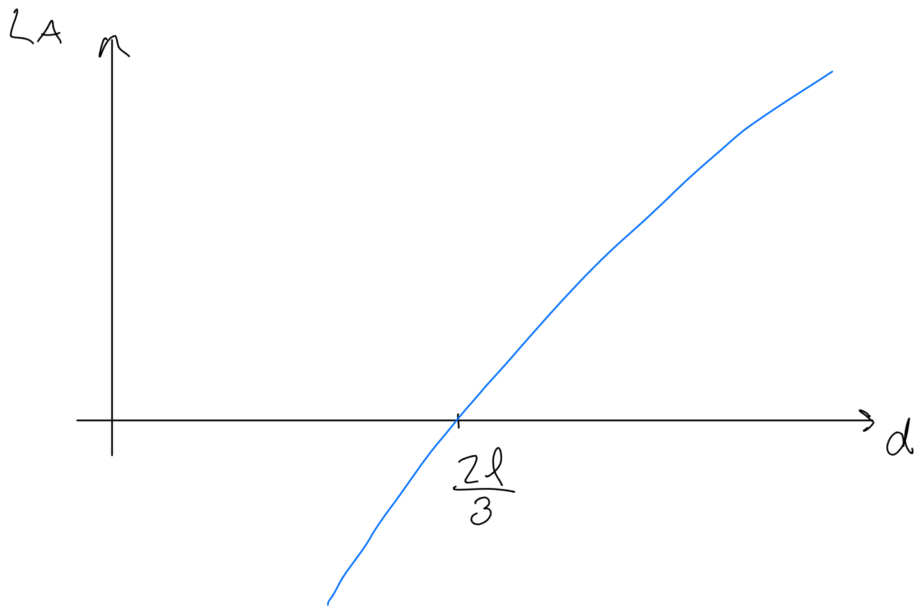
$$\circlearrowleft A: \; -L_B \cdot d = -\underbrace{I_A}\omega_e - I_A\omega \quad (2)$$

$$\frac{m\ell^2}{12} + m\left(\frac{\ell}{2}\right)^2 = \frac{m\ell^2}{3}$$

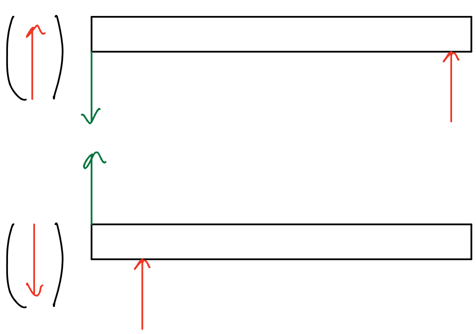
(2) $\Rightarrow$

$$d = \frac{\frac{ml^2}{3} \cdot (\omega_e + \omega)}{L_B} = \frac{\frac{ml^2}{3} \cdot (\omega_e + \omega)}{m\frac{l}{2} \; (\omega_e + \omega)} = \frac{2l}{3}$$

B kallas då perfuisionscentrum/sweeeet spot.



Ex: Penna



Enkel stöt mellan två kroppar

$$e = \frac{(V_{s_1}^{e} - V_{s_2}^{e})\,\hat{n}}{(V_{s_2}^{f} - V_{s_1}^{f})\,\hat{n}}$$