Lektion 9

TAMS24 – Statistisk teori

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Lektionsgenomgång $Q_1 - Q_5$

G 2.3

	i	$\hat{\beta}_i$	$d(\hat{\beta}_i)$	
β_0	0	$-0.028..$	$\hat{\beta}_0$ $0.079..$	$= d(\hat{\beta}_0)$
β_1	1	$0.55...$	$\hat{\beta}_1$ 0.033	$= s\sqrt{h_{00}}$
β_2	2	$-0.047..$	$\hat{\beta}_2$ 0.002	$= d(\hat{\beta}_1) = s\sqrt{h_{11}}$

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$$

$$\hat{\mu} = y = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2$$

$$s^2 = \frac{SS_E}{n-k-1}$$

$$\mathbb{X} = \begin{pmatrix} 1 & x_{11} & \cdots & x_{1k} \\ 1 & x_{21} & & x_{2k} \\ \vdots & \vdots & & \\ 1 & x_{n1} & & x_{nk} \end{pmatrix}$$

$$(\mathbb{X}^T \mathbb{X})^{-1} = \begin{pmatrix} h_{00} & h_{01} \cdots & h_{0k} \\ h_{10} & h_{11} \cdots & h_{1k} \\ \vdots & & \\ h_{k0} & & h_{kk} \end{pmatrix}$$

<u>14.7</u>

$$Y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$$

$$\varepsilon \sim N(0, \sigma)$$

a) $H_{\beta_1}: \beta_1 = 0, \quad H_{\beta_2}: \beta_2 = 0$

$$\begin{cases} H_0: B_1 = 0 \\ H_1: B_1 \neq 0 \end{cases}$$

Fact:

$$\frac{\hat{B}_1 - B_1}{s\sqrt{h_{11}}} \sim t(n-k-1)$$

$\hat{B}_1 \sim N(\hat{B}_1, \sigma\sqrt{h_{11}})$, $\frac{\hat{B}_1 - B_1}{\sigma\sqrt{h_{11}}} \sim N(0,1)$

p-value < α, reject H_0

$8{,}3 \cdot 10^{-6} < 0.05 \Rightarrow$ reject H_0

$$\begin{cases} H_0: B_2 = 0 \\ H_1: B_2 \neq 0 \end{cases}$$

$0.13 \not< 0.05 \Rightarrow$ don't reject H_0.

b) $Y = \hat{\mu} = \alpha + \hat{\beta}_1 x_1 + \hat{\beta}_2 \times x_2 + \varepsilon$

$E(Y) = \alpha + B_1(x_1 + 1) + B_2(x_2 + 1) \Rightarrow \hat{B}_1 + \hat{B}_2 = \Delta$

$\hat{B}_1 + \hat{B}_2 = 1.423 - 0.176 = 1.247$

c) $V(\hat{B}_1 + \hat{B}_2) = V(\hat{B}_1) + V(\hat{B}_2) + 2C(\hat{B}_1, \hat{B}_2) =$

$= 0.247^2 + 0.112^2 + 2(-0.00248) = 0.0609\ldots$

$\underline{d(\hat{B}_1, \hat{B}_2)} = \sqrt{0.0609\ldots} \approx 0.25$

G.2.3

Fact:

$$\frac{SSR/k}{SSE/(n-k-1)} \sim F(k, n-k-1)$$

$$TS = \frac{SSR/k}{SSE/(n-k-1)} = \frac{1.26130/2}{0.03186/7} = 138.56$$

$$C = (F_\alpha(k, n-k-1), \infty) = F_{0.01}(2,7) = 21.96$$

If $TS \in C$, recject H_0.

$TS \in C \Rightarrow$ reject H_0.

b) $\begin{cases} H_0 : B_1, B_2 = 0 \\ H_1 : B_1, B_2 \neq 0 \end{cases}$

$$y' = 0.551 - 2 \cdot 0.0473x = 0 \iff x = \frac{0.551}{2 \cdot 0.0473} = 5.82$$

c) $\begin{cases} H_0 : B_j = 0 \\ H_1 : B_j \neq 0 \end{cases}$

$$TS = \frac{\hat{B}_j}{s\sqrt{n_{jj}}} = \frac{\hat{B}_j}{\sigma(\hat{B}_j)} = \begin{cases} j = \cancel{0} \\ j = 1 : & 16.64 \\ j = 2 : & -16.64 \end{cases}$$

$$C = (-\infty, -t_{0.0005}\underbrace{(n-k-1)}_{7}) \cup (t_{0.005}\underbrace{(n-k-1)}_{7}, \infty)$$

$TS \in C \Rightarrow$ **reject H_0**

$\Rightarrow C = (-\infty, -4.41) \cup (4.41, \infty)$