# Föreläsning 16

TMME04 – Mekanik II

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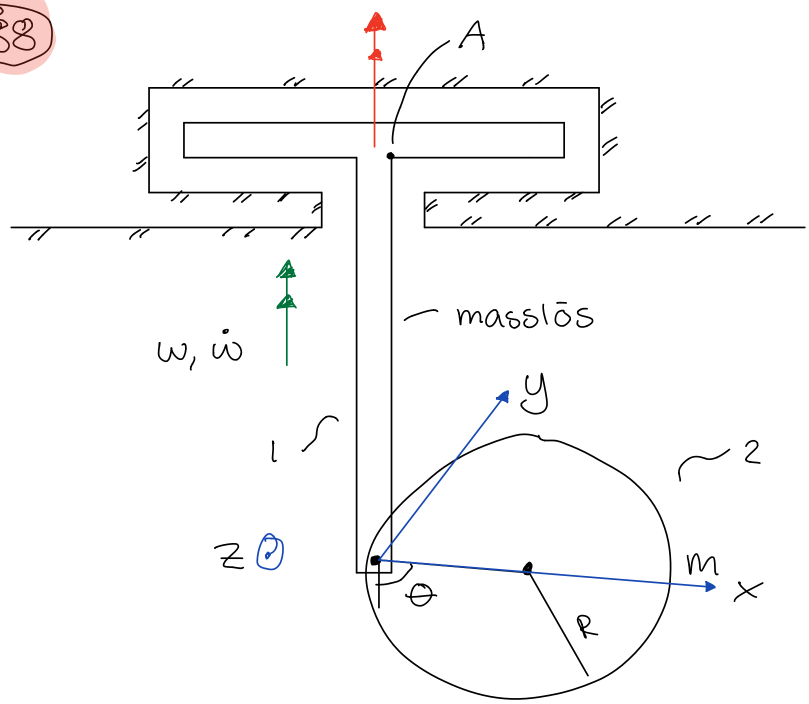
88




Sökt: rörelseekvationerna

Euler II, för 1+2:

$$\bar{M}_O^{ext} = \underbrace{\dot{\bar{h}}_O^{(1)}}_{\substack{=0 \\ \text{ty masslös}}} + \dot{\bar{h}}_O^{(2)} = \left(\frac{d\bar{h}_O^{(2)}}{dt}\right)_r + \bar{\omega}_r \times \bar{h}_O^{(2)}, \qquad (1)$$

O fix i i-ram

$$\bar{h}_O^{(2)} = I_O^{(2)} \bar{\omega}_2 ,$$

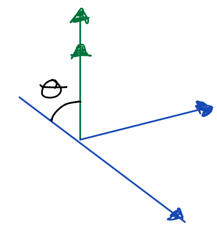
O fix i kropp 2.

Val av referensram att derivera i:

$$r = 2$$

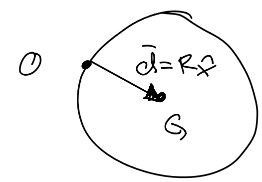
Inför Oxyz fixt i kropp 2

$$\bar{\omega}_2 = \bar{\omega}_{2/1} + \bar{\omega}_{1/0} = \dot{\theta}\,\hat{z} + \omega(-\cos\theta\,\hat{x} + \sin\theta\,\hat{y})$$




$$\bar{h}_O^{(2)} = \begin{bmatrix} I_{Oxx} & 0 & 0 \\ 0 & I_{Oyy} & 0 \\ 0 & 0 & I_{Oxx} \end{bmatrix} \begin{bmatrix} -\omega\cos\theta \\ \omega\sin\theta \\ \dot{\theta} \end{bmatrix} \qquad (2)$$

Huygens sats:




$$I_{Oxx} = \underbrace{I_{Gxx}}_{\frac{mR^2}{4}} + m(d_y^2 + d_z^2) = \frac{mR^2}{4}$$

$$I_{Oyy} = \underbrace{I_{Gyy}}_{\frac{mR^2}{4}} + m(d_x^2 + d_z^2) = \frac{5mR^2}{4}$$

$$I_{Ozz} = \underbrace{I_{Gzz}}_{\frac{mR^2}{2}} + m(d_x^2 + d_y^2) = \frac{3mR^2}{2}$$

Insättning i (2) =>

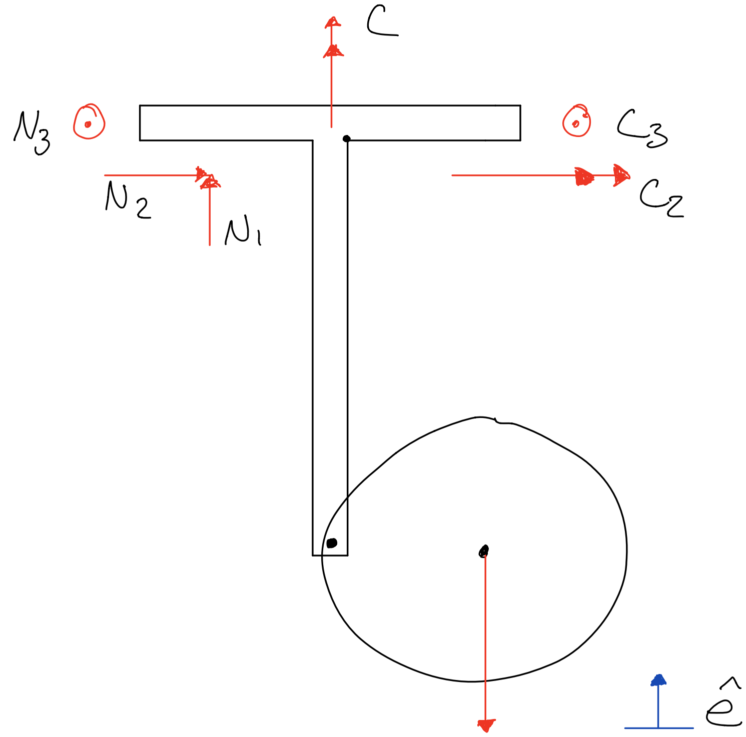
$$\bar{h}_O^{(2)} = mR^2\left(-\frac{1}{4}\omega\cos\theta\,\hat{x} + \frac{5}{4}\omega\sin\theta\,\hat{y} + \frac{3}{2}\dot{\theta}\,\hat{z}\right)$$

$$\left(\frac{d\bar{h}_O^{(2)}}{dt}\right)_2 = mR^2\left[\left(-\frac{1}{4}\dot{\omega}\cos\theta + \frac{1}{4}\omega\dot{\theta}\sin\theta\right)\hat{x} + \left(\frac{5}{4}\dot{\omega}\sin\theta + \frac{5}{4}\omega\dot{\theta}\cos\theta\right)\hat{y} + \frac{3}{2}\ddot{\theta}\,\hat{z}\right]$$

Insättning i (1) =>

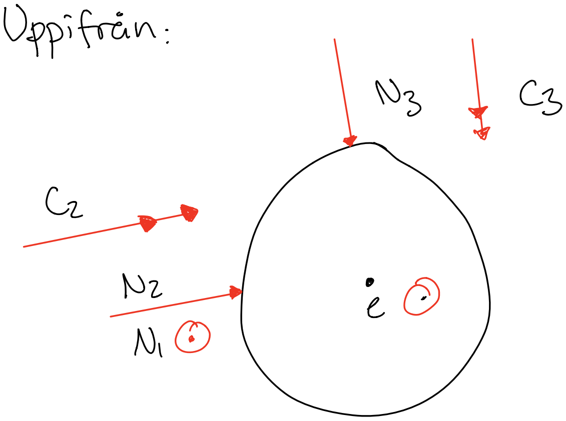
$$\bar{M}_O^{ext} = mR^2\left[\left(-\frac{1}{4}\dot{\omega}\cos\theta + \frac{1}{2}\omega\dot{\theta}\sin\theta\right)\hat{x} + \left(\frac{5}{4}\dot{\omega}\sin\theta + \frac{5}{2}\omega\dot{\theta}\cos\theta\right)\hat{y} + \left(\frac{3}{2}\ddot{\theta} - \omega^2\sin\theta\cos\theta\right)\hat{z}\right] \qquad (3)$$

Fri lägg, kropp 1+2




$\hat{e} = -\cos\theta\,\hat{x} + \sin\theta\,\hat{y}$

Uppifrån:



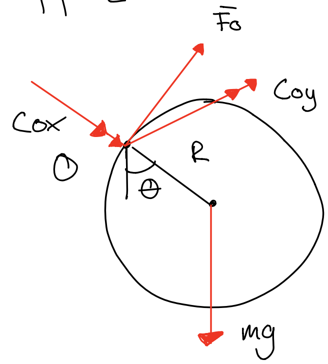

Momentet kring $\hat{e}$-axeln är $C$ (ty $mg \parallel \hat{e}$, $N_1 \parallel \hat{e}$, $N_2, N_3$ skär $\hat{e}$).

$$C = M_O^{ext} \cdot \hat{e}$$

(3) $\Rightarrow$

$$C = mR^2 \left(2\dot{\theta}\omega \sin\theta\cos\theta + \frac{1}{4}\dot{\omega} + \dot{\omega}\sin^2\theta\right)$$

Frilägg, kropp 2



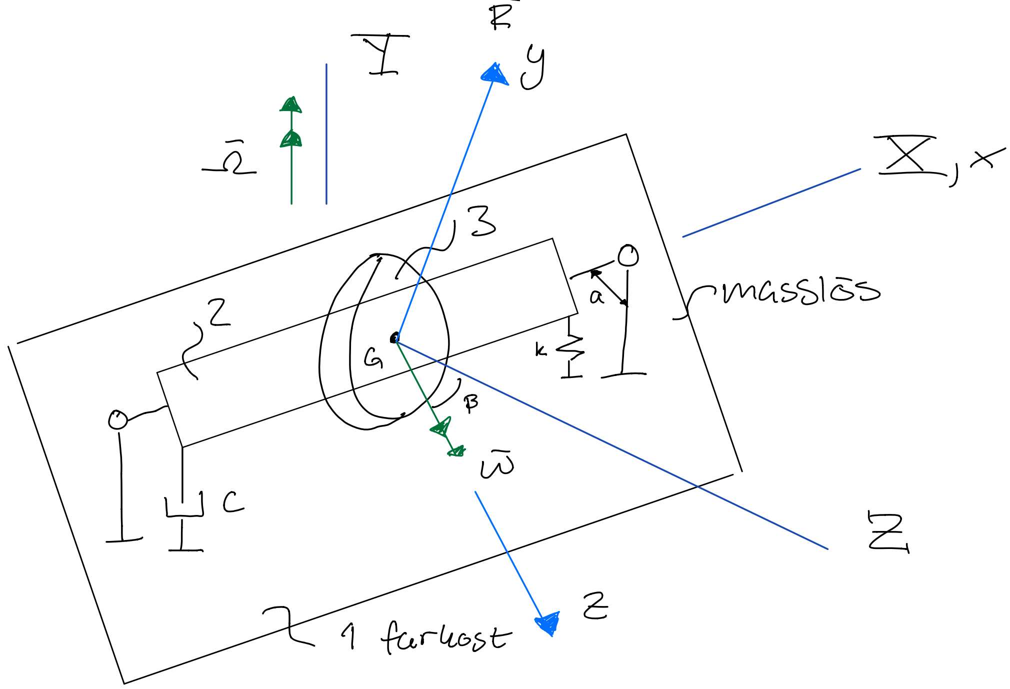

$\bar{M}_O = \dot{\bar{h}}_O$ ②

$M_{Oz} = -R\sin\theta\, mg$

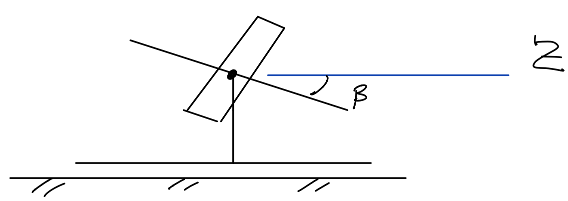
Insättning i (3) ⇒

$$\frac{3}{2}\ddot{\theta} - \omega^2 \sin\theta\cos\theta = -\frac{g}{R}\sin\theta$$

87




O - marken



Givet: $\omega$ konst

$\beta$ liten

$\omega >> \Omega$

Sökt: Differentialekvation för $\beta$

$\beta$ då $t \to \infty$, $\Omega$ konstant

Euler II rotor+ram (masslös)

$$\bar{M}_G^{ext} = \bar{h}_G^{(3)} =$$ ty ramen masslös

$$= \left(\frac{d\bar{h}_G^{(3)}}{dt}\right)_r + \bar{\omega}_r \times \bar{h}_G \qquad (1)$$

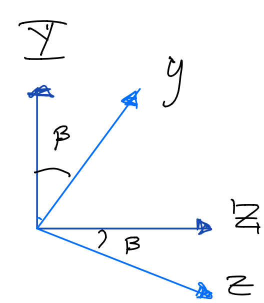
$$\bar{h}_G^{(3)} = I_G^{(3)} \bar{\omega}_3$$

Val av referensram att derivera i:

$$r = 2$$

ger $I_G^{(3)}$ konstant, pga rotationssymmetri.

Inför $Gxyz$ fixt i ramen




$$\bar{\omega}_2 = \bar{\omega}_{2/1} + \bar{\omega}_{1/0}$$

$$\bar{\omega}_3 = \bar{\omega}_{3/2} + \bar{\omega}_2$$

$$\bar{\omega}_{1/0} = \Omega \hat{Y} = \Omega(\cos\beta\, \hat{y} - \sin\beta\, \hat{z})$$

$$\bar{\omega}_{2/1} = \dot{\beta}\, \hat{x}$$

$$\omega_{3/2} = \omega \hat{z}$$

$$\therefore \bar{\omega}_2 = \dot{\beta}\, \hat{x} + \Omega\cos\beta\, \hat{y} - \Omega\sin\beta\, \hat{z}$$

$$\bar{\omega}_3 = \dot{\beta}\,\hat{x} + \Omega\cos\beta\,\hat{y} + (\omega - \Omega\sin\beta)\,\hat{z}$$

$$\bar{h}_G^{(3)} = \begin{bmatrix} J & 0 & 0 \\ 0 & J & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \dot{\beta} \\ \Omega\cos\beta \\ \omega - \Omega\sin\beta \end{bmatrix} =$$

"$I \approx J$"

$$= J\dot{\beta}\,\hat{x} + J\Omega\cos\beta\,\hat{y} + I(\omega - \Omega\sin\beta)\,\hat{z}$$

$$\left(\frac{d\bar{h}_G^{(3)}}{dt}\right)_2 = J\ddot{\beta}\,\hat{x} + J\dot{\Omega}\cos\beta\,\hat{y} - J\Omega\sin\beta\dot{\beta}\,\hat{y} -$$

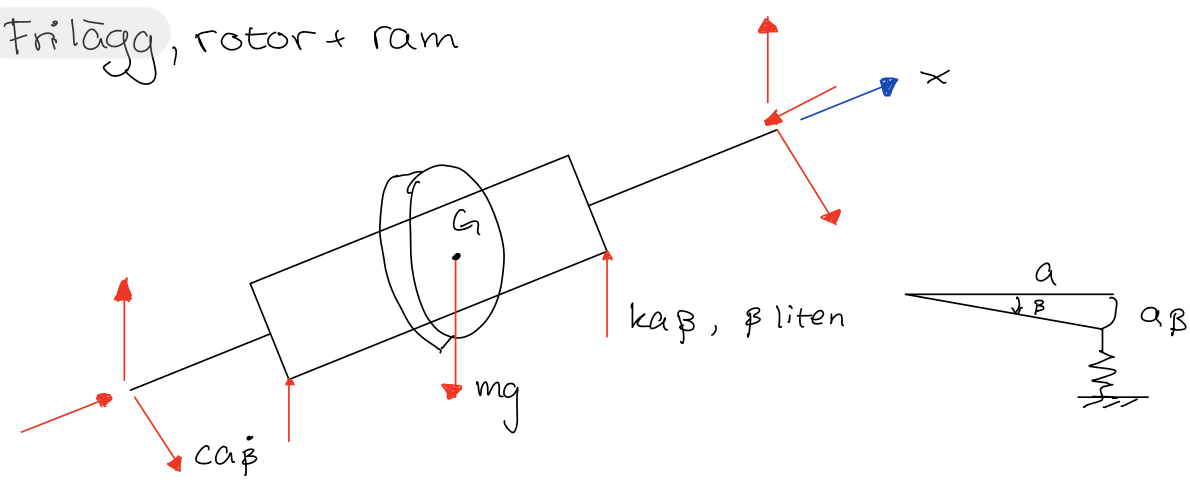
$$- I\dot{\Omega}\sin\beta\,\hat{z} - I\Omega\cos\beta\dot{\beta}\,\hat{z}$$

Insättning i (1) $\Rightarrow$

$$\Big/\ \begin{matrix} \Omega \ll \omega \\ J\Omega \ll I\omega \end{matrix}\ \Big/ \Rightarrow$$

$$\bar{M}_G^{ext} = (J\ddot{\beta} + I\Omega\omega\cos\beta)\,\hat{x} + (J\dot{\Omega}\cos\beta - I\omega\dot{\beta})\,\hat{y}$$
$$- (I\dot{\Omega}\sin\beta + I\Omega\cos\beta\dot{\beta})\,\hat{z} \quad (2)$$

Frilägg, rotor + ram



$$M_{G,x}^{ext} = -ka^2\beta - ca^2\dot{\beta}$$

(2) $\Rightarrow$

$$J\ddot{\beta} + I\Omega\omega\cos\beta = -ka^2\beta - ca^2\dot{\beta} \Leftrightarrow$$
$$J\ddot{\beta} + ca^2\dot{\beta} + ka^2\beta = -\omega I\Omega$$

Differentialekvation för dämpad svängning!

$\Omega$ konstant: "$t \to \infty$" $\Rightarrow$ $\beta = \frac{-\omega I}{ka^2}\Omega$

Rategyro: läser av $\beta \Rightarrow$ för $\Omega$

$\underline{\underline{k=0}}$ $\Rightarrow$

$$J\ddot{\beta} + ca^2\dot{\beta} = -\omega I\Omega \Rightarrow / \omega, \Omega \text{ konstant} / \Rightarrow$$
$$J\dot{\beta} + ca^2\beta = -\omega I\Omega t$$

"$t \to \infty$" $\Rightarrow$

$$\beta = -\frac{\omega I}{ca^2}\Omega t + \underbrace{\text{konstant}}_{\approx 0}$$

Vinkelmätande gyro (rate integrating gyro)