Föreläsning 13

TMME04 – Mekanik II

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Euler II, godtycklig punkt

$$\bar{M}_A = \bar{h}_G + \bar{r}_{AG} \times m\bar{a}_G$$

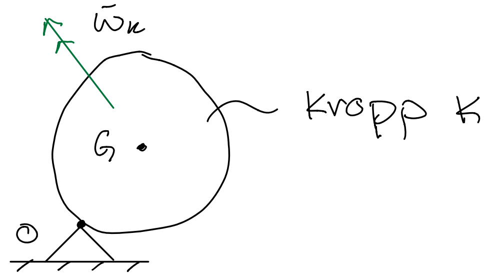
$$\boxed{\bar{M}_A = \left(\frac{d\bar{h}_G}{dt}\right) + \bar{\omega}_r \times \bar{h}_G + \bar{r}_{AG} \times m\bar{a}_G}$$

Euler II, punkt O fix i i-ram och kropp

samma koordinat hela tiden

$$\bar{M}_O = \dot{\bar{h}}_O$$

$$\boxed{M_O = \left(\frac{d\bar{h}_O}{dt}\right)_r + \omega_r \times \bar{h}_O}$$

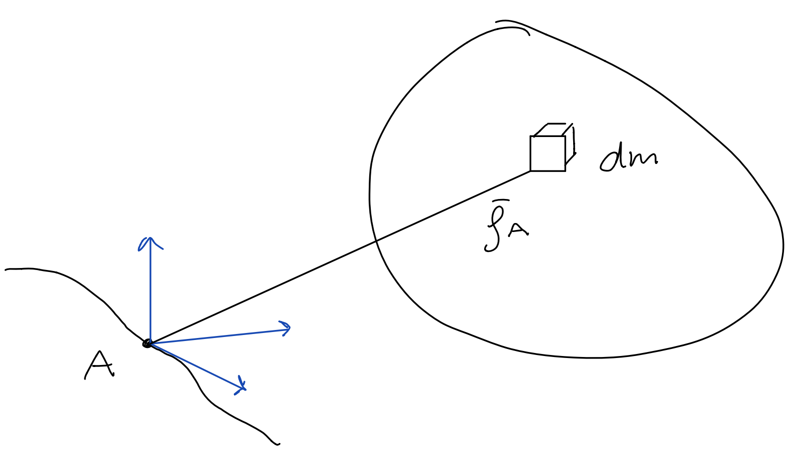


där

$$[h_O] = [I_{\bar{O}}][\bar{\omega}_K]$$

$[I_{\bar{O}}]$ ges av (4) nedan.

Masströghetsmatrisen



A godtycklig punkt.

Def:

(Mass)tröghetsmatrisen map koordinatsystemet $Axyz$,

$$\boxed{[\bar{I}_A] = \int \left(\rho_A^2 [\bar{E}] - [\bar{\rho}_A][\bar{\rho}_A]^T \right) dm}$$

Explicit:

$$[\bar{\rho}_A] = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \rho_A^2 [\bar{E}] - [\bar{\rho}_A][\bar{\rho}_A]^T =$$

$$= (x^2 + y^2 + z^2) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} x & y & z \end{bmatrix} =$$

$$= \begin{bmatrix} y^2 + z^2 & -xy & -xz \\ -yx & x^2 + z^2 & -yz \\ -zx & -zy & x^2 + y^2 \end{bmatrix} \Rightarrow$$

$$[\bar{I}_A] = \begin{bmatrix} I_{Axx} & I_{Axy} & I_{Axz} \\ I_{Axy} & I_{Ayy} & I_{Ayz} \\ I_{Axz} & I_{Azy} & I_{Azz} \end{bmatrix}$$

där

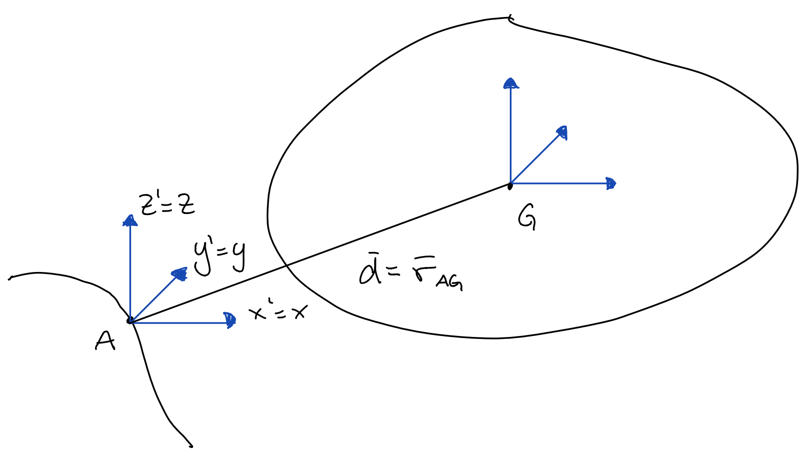
Masströghetsmoment   Masströghetsprodukter

$$I_{Axx}=\int(y^2+z^2)\,dm,\quad I_{Axy}=-\int xy\,dm$$

$$I_{Ayy}=\int(x^2+y^2)\,dm,\quad I_{Axz}=-\int xz\,dm$$

$$I_{Azz}=\int(x^2+y^2)\,dm,\quad I_{Ayz}=-\int yz\,dm$$

$[\bar{I}_A]$ är symmetrisk, beror på valet av koordinatsystem.



A godtycklig punkt

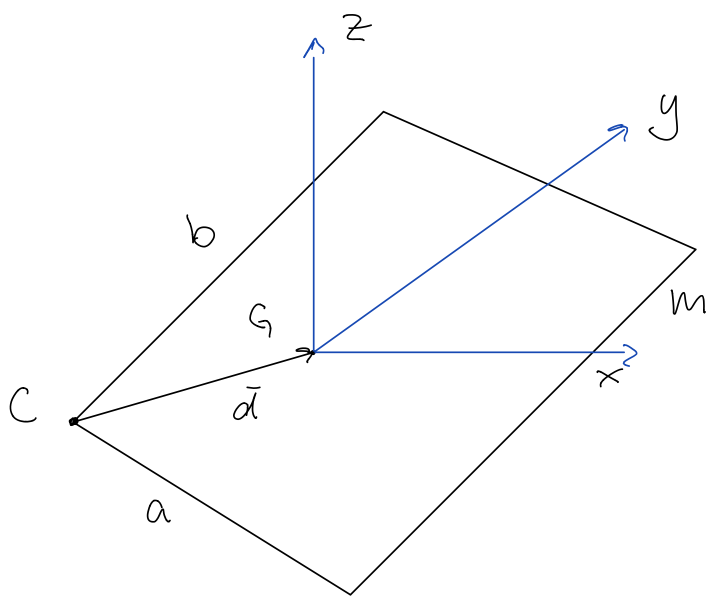
SATS: Huygens Sats:

$$[\bar{I}_A]=[\bar{I}_G]+m\left(d^2[\bar{E}]-[\bar{d}][\bar{d}]^T\right) \quad (4)$$

Explicit att:

$$\boxed{\begin{aligned}
I_{Axx} &= I_{Gxx} + m(d_y^2 + d_z^2)\\
I_{Ayy} &= I_{Gyy} + m(d_x^2 + d_z^2)\\
I_{Azz} &= I_{Gzz} + m(d_x^2 + d_y^2)\\
I_{Axy} &= I_{Gxy} - m d_x d_y\\
I_{Axz} &= I_{Gxz} - m d_x d_z\\
I_{Ayz} &= I_{Gyz} - m d_y d_z
\end{aligned}}$$

Ex: Tunn skiva

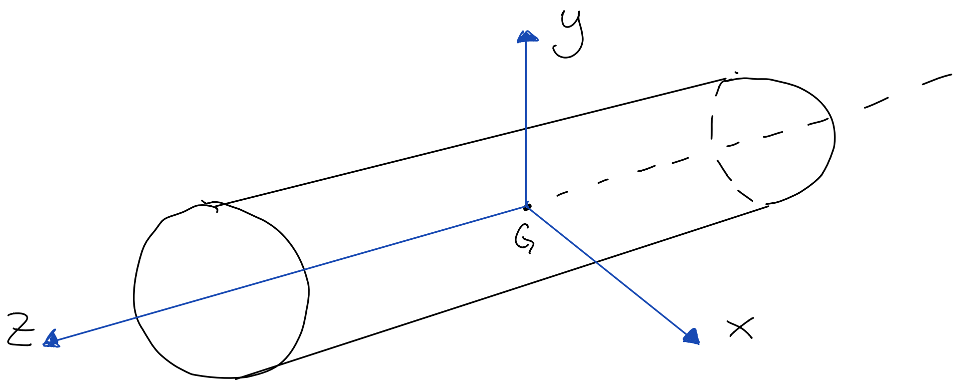


$$\bar{d} = \frac{a}{2}\hat{x} + \frac{b}{2}\hat{y}$$

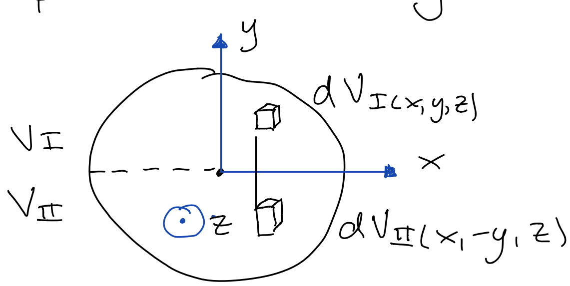
$$I_{Cxx} = \underbrace{I_{Gxx}}_{\frac{mb^2}{12}} + m(d_y^2 + d_z^2) = \frac{mb^2}{3}$$

Tröghetsprodukter, vid symmetri

Ex: Homogen cylinder



xz-planet är ett symmetriplan:



$$I_{cxy} = -\int xy\rho \, dV = -\underbrace{\int_{V_I} xy\rho \, dV_I}_{\int x(-y)\rho \, dV_{II}} - \int_{V_{II}} xy\rho \, dV_{II} =$$

$$= 0$$

P.S.S

$$I_{cyz} = 0.$$

Allmänt för $Cxyz$:

xy-planet symmetriplan $\Rightarrow$

$$I_{Cxz} = I_{Cyz} = 0$$

xz-planet symmetriplan $\Rightarrow$

$$I_{Cxy} = I_{Cyz} = 0$$

yz-planet symmetriplan $\Rightarrow$

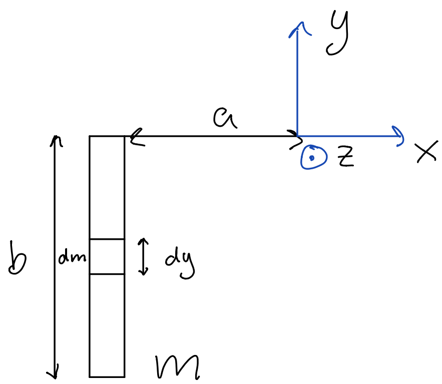
$$I_{Cxy} = I_{Cxz} = 0$$

Ex: Tunn skiva, fortsättning

$$I_{Cxy} = \underbrace{I_{Gxy}}_{=0} - m d_x d_y = -\frac{mab}{4}.$$

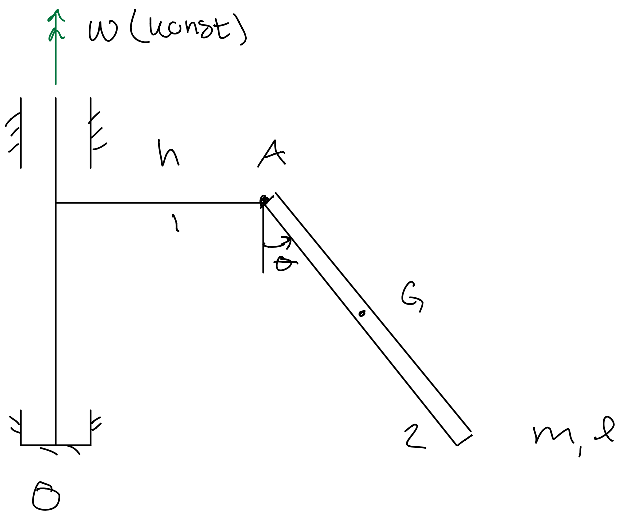
ty yz sym.plan

Ex:



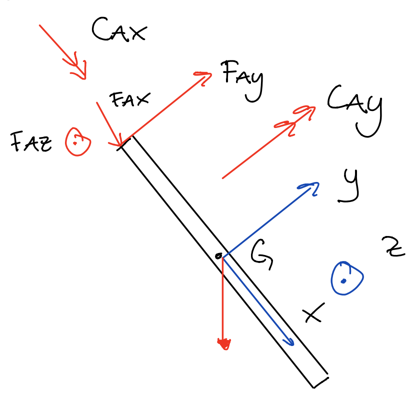
$$I_{Oxy} = -\int xy\,dm = -\int_{-b}^{0} \underbrace{x}_{-a} y\, m \frac{dy}{b} = -\frac{mab}{2}.$$

Ex:



Sökt: ω så θ konstant

Frilägg:



Euler II

$$\bar{M}_A = \left(\frac{d\bar{h}_G}{dt}\right)_r + \bar{\omega}_r \times \bar{h}_G + \bar{r}_{AG} \times m\bar{a}_G \quad (1)$$

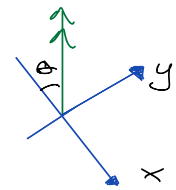
$$\bar{h}_G = \bar{I}_G \bar{\omega}_2$$

Derivera i kropp r

$$r = 1 \quad (= 2)$$

Gxyz fixt i kropp 1 (= 2)

$\bar{\omega}_1 = \omega(-\cos\theta\,\hat{x} + \sin\theta\,\hat{y})$

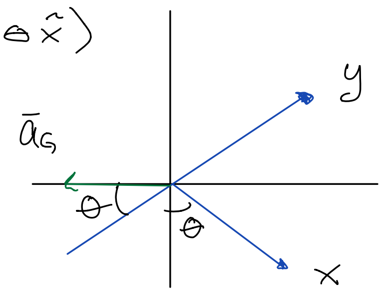


$\bar{\omega}_2 = \bar{\omega}_{2/1} + \bar{\omega}_{1/0} = \bar{\omega}_1$

$$\bar{h}_G = \begin{bmatrix} 0 & 0 & * \\ 0 & ml^2/12 & * \\ 0 & 0 & * \end{bmatrix} \begin{bmatrix} -\omega\cos\theta \\ \omega\sin\theta \\ 0 \end{bmatrix} = \frac{ml^2}{12}\omega\sin\theta\,\hat{y}$$

$$\left(\frac{d\bar{h}_G}{dt}\right)_1 = \bar{0}$$

$\bar{a}_G = \left(h + \frac{l}{2}\sin\theta\right)\omega^2(-\cos\theta\,\hat{y} - \sin\theta\,\hat{x})$



H.L i (1) blir

$$\bar{M}_A = -\frac{ml^2}{12}\omega^2\sin\theta\cos\theta\,\hat{z} -$$

$$-\frac{ml\omega^2}{2}\left(h + \frac{l}{2}\sin\theta\right)\cos\theta\,\hat{z}$$

Men

$$\bar{M}_A = -mg\frac{l}{2}\sin\theta\,\hat{z} + C_{Ax}\hat{x} + C_{Ay}\hat{y}$$

Identifiering $\Rightarrow \omega$.