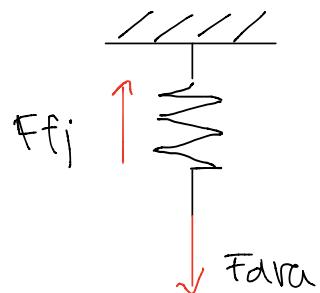
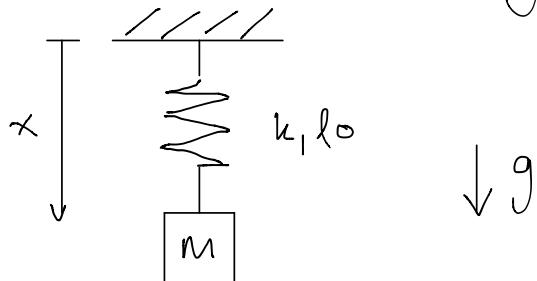


Föreläsning 7

TMME12 – Mekanik I

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Fri odämpad svängning



- Fri laggnings:

$$\begin{array}{c} \uparrow \\ \text{---} \\ \uparrow \\ \text{mg} \end{array} \quad k(x - l_0)$$

- Newton II

$$\downarrow : mg - k(x - l_0) = m\ddot{x} \Leftrightarrow \text{Standard form; } x, \ddot{x}$$

på samma sida $\Leftrightarrow \ddot{x} + \frac{k}{m}x = g + \frac{k l_0}{m}$ (1)

$\underbrace{\quad}_{\omega_n^2}$

ω_n : egenvinkelfrekvens

$$x = x_h + x_p$$

- Homogen lösning x_h :

$$x_h = A \cos \omega_n t + B \sin \omega_n t$$

- Partikulär lösningen X_p :

$X_p = C$, konstant

$$(1) \Rightarrow \frac{k}{m}C = g + \frac{kl_0}{m}$$

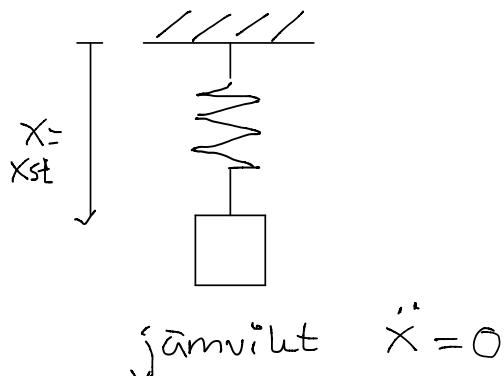
$$\Leftrightarrow C = \frac{mg}{k} + l_0$$

$$\therefore X = A \cos \omega_n t + B \sin \omega_n t + \frac{mg}{k} + l_0$$

- Bestäm A & B från givena $x(0)$ & $\dot{x}(0)$.
- Svängningstid T_n :

$$\omega_n T_n = 2\pi \Leftrightarrow T_n = \frac{2\pi}{\omega_n}$$

- Bestämning av jämviktsläget, x_{st} : statisk



$$(1) \Rightarrow \frac{k}{m} x_{st} = g + \frac{kl_0}{m} \Leftrightarrow x_{st} = l_0 + \frac{mg}{k}$$

Fri dämpad svängning

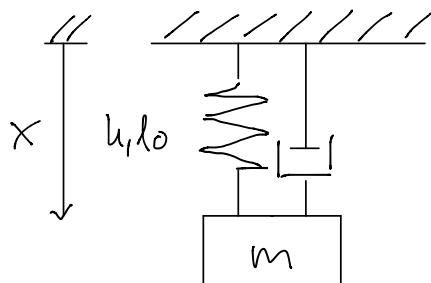
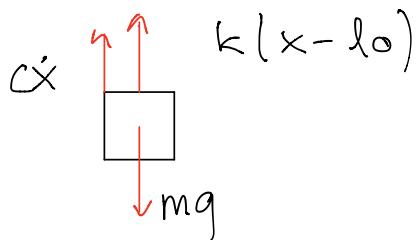


Fig 1.

- Fri läggning:



- Newton II

$$\downarrow : mg - k(x - l_0) - c\dot{x} = m\ddot{x}$$

$$\Leftrightarrow \ddot{x} + \underbrace{\frac{c}{m}\dot{x}}_{2\zeta\omega_n} + \underbrace{\frac{k}{m}x}_{\omega_n^2} = g + \frac{k l_0}{m} \quad (2)$$

ζ är dämpfaktorn

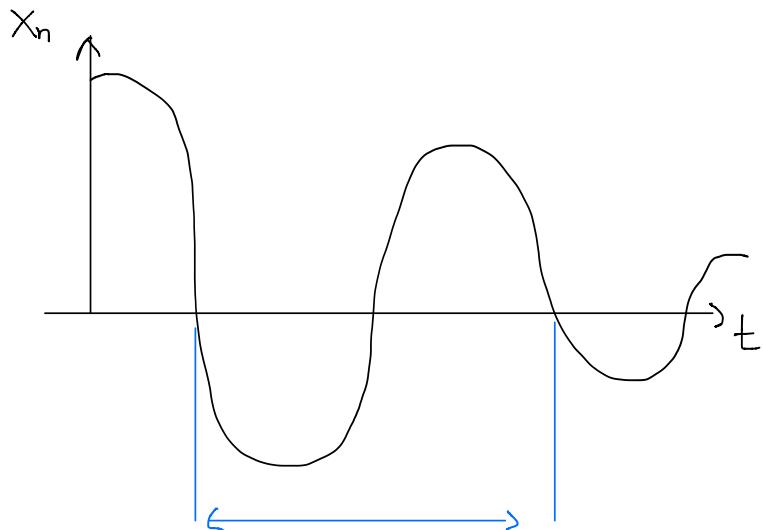
- $X = X_h + X_p$

$\zeta < 1$: underdämppat system

$$X_h = e^{-\zeta\omega_n t} (A \cos \omega_d t + B \sin \omega_d t)$$

$$\omega_d = \sqrt{1 - \zeta^2} \omega_n$$

ω_d dämpad egenvibrationsfrekvens



$$\tilde{\tau}_d = \frac{2\pi}{\omega_d} \rightarrow \tilde{T}_n !$$

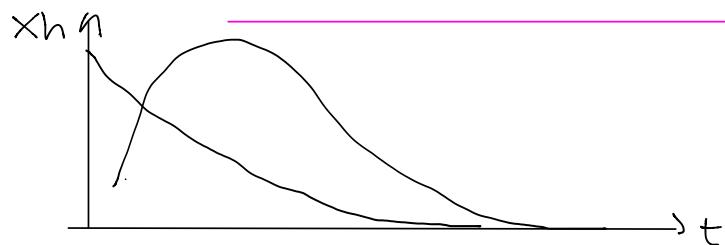
- Fortvarighet

$$x_n = 0, \text{ då } x = x_p$$

Lösningem åberoende av begynnelsevilkuren.

$\zeta > 1$: Överdämpat system

$$x_h = A e^{-(\zeta - \sqrt{\zeta^2 - 1}) \omega_n t} + B e^{-(\zeta + \sqrt{\zeta^2 - 1}) \omega_n t}$$



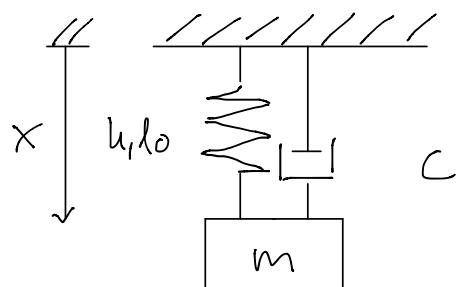
$\zeta = 1$: Kritiskt dämpat system

$$x_h = (A \cdot t + B) e^{-\omega_n t}$$



Konvergenan snabbare än för överdämpat.

Ex:

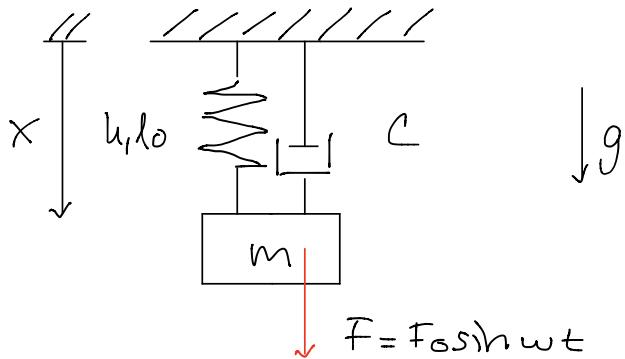


$G: k, m, \zeta = 1$

$S: C$ -dämpningskonstant

$$[2] \Rightarrow 2\zeta\omega_n = \frac{c}{m} \Leftrightarrow c = 2\sqrt{mk}$$

Påtvingad dämpad svängning



- Newton II:

$$\downarrow : mg - k(x - x_0) - C\dot{x} + F_0 \sin \omega t = m\ddot{x}$$

$$\Leftrightarrow \ddot{x} + \underbrace{\frac{C}{m}\dot{x}}_{2\zeta\omega_n} + \underbrace{\frac{k}{m}x}_{\omega^2} = g + \frac{kx_0}{m} + \frac{F_0}{m} \sin \omega t \quad (3)$$

- $X = X_h + X_p$

Vid fortvarighet $X = X_p$ ($X_h = 0$)

$$X_p = \underbrace{X_1}_{=0} + \underbrace{X_2}_{=0} \sin \omega t + \underbrace{X_3}_{=0} \cos \omega t$$

$= 0$ om $\zeta = 0$, odämpat

$$\dot{X}_p = \underbrace{X_2}_{=0} \omega \cos \omega t - \underbrace{X_3}_{=0} \omega \sin \omega t$$

$$\ddot{X}_p = -\underbrace{X_2}_{=0} \omega^2 \cos \omega t - \underbrace{X_3}_{=0} \omega^2 \sin \omega t$$

Insättning i (3) \Rightarrow

$$\begin{aligned}
 & -\sum_2 \omega^2 \sin \omega t - \sum_3 \omega^2 \cos \omega t + \frac{C}{m} (\sum_2 \omega \cos \omega t - \\
 & - \sum_3 \omega \sin \omega t) + \frac{k}{m} (\sum_1 + \sum_2 \cdot \sin \omega t + \sum_3 \cos \omega t) = \\
 & = g + \frac{h \omega_0}{m} + \frac{F_0}{m} \sin \omega t
 \end{aligned}$$

• Identifiera:

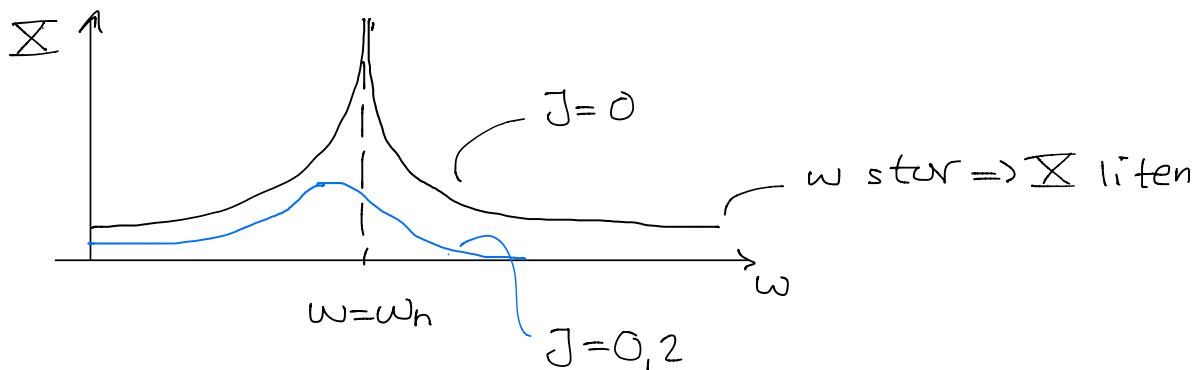
Konstant: $\frac{k}{m} \sum_1 = g + \frac{h \omega_0}{m}$

$\sin \omega t: -\sum_2 \omega^2 - \frac{C}{m} \sum_3 \omega + \frac{k}{m} \sum_2 = \frac{F_0}{m}$

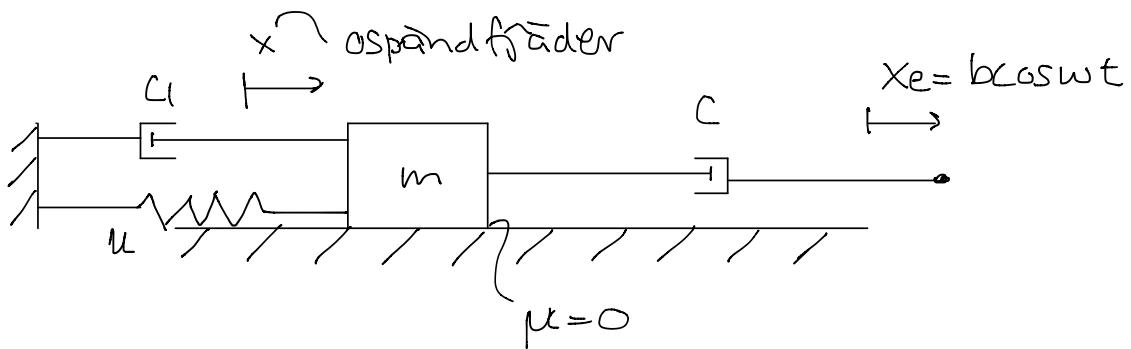
$\cos \omega t: -\sum_3 \omega^2 + \frac{C}{m} \sum_2 \omega + \frac{k}{m} \sum_3 = 0$

$\Rightarrow \sum_1, \sum_2, \sum_3$ kan släcka

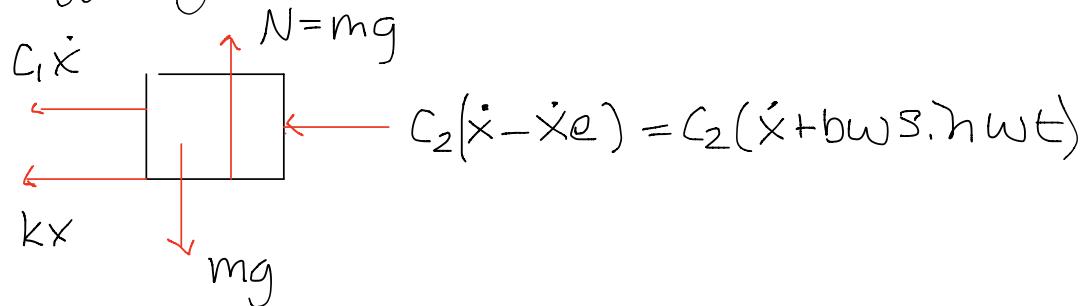
$$\begin{aligned}
 X_p &= \sum_1 + \sum \sin(\omega t + \varphi) \\
 &\quad \text{amplitud} \qquad \text{fasvinkel} \\
 &= \sqrt{\sum_2^2 + \sum_3^2}
 \end{aligned}$$



Ex:



• Friläggning:



$$\rightarrow -kx - C_1 \dot{x} - C_2 (\dot{x} + b \cos \omega t) = m \ddot{x}$$

$$\Leftrightarrow \ddot{x} + \underbrace{\frac{C_1 + C_2}{m} \dot{x}}_{2\zeta\omega_n} + \underbrace{\frac{k}{m} x}_{\omega_n^2} = -\frac{b\omega C_2}{m} \sin \omega t$$