Lösningsgång

TAOP07 – Optimeringslära grundkurs

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Produkter: P_1, P_2, P_3

Vinst: 25, 30, 20 kr

Resurs: R_1: 50, 24, 25

R_2: 30, 33, 15

Mängd: R_1: 2400

R_2: 2100

x_i = pris som företag B betalar per enhet av resurs R_j, $j = 1,2$

$$\min \; 2400x_1 + 2100x_2$$

då

$$50x_1 + 30x_2 \geq 25$$
$$24x_1 + 33x_2 \geq 30$$
$$25x_1 + 15x_2 \geq 20$$
$$x_1, x_2 \geq 0$$

2

$$\min \; 2x_4 + 5x_5$$

(P)

$$\begin{array}{lll} \text{då} & 2x_1 + 3x_2 + 4x_3 - x_4 = 2 & \Big| \; y_1 \\ & x_1 + 2x_2 + 3x_3 - x_5 = 3 & \Big| \; y_2 \\ & x_1, \dots, x_5 \geq 0 & \end{array}$$

(D)

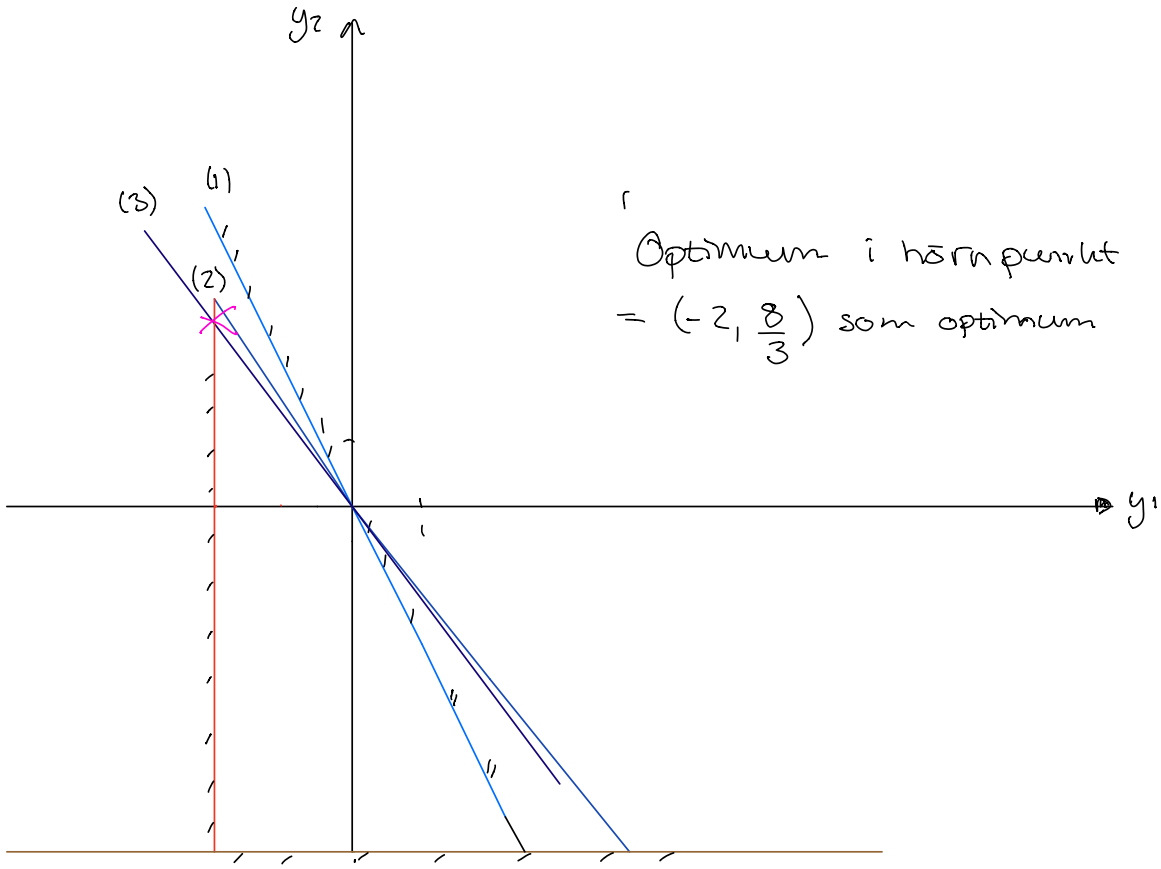
$$\max \; 2y_1 + 3y_2$$

$$\begin{array}{lll} \text{då} & 2y_1 + y_2 \leq 0 & (1) \\ & 3y_1 + 2y_2 \leq 0 & (2) \\ & 4y_1 + 3y_2 \leq 0 & (3) \end{array}$$

$-y_1 \le 2 \qquad (4)$

$-y_2 \le 5 \qquad (5)$

y_1, y_2 fri



Optimum i hörnpunkt

$= (-2, \frac{8}{3})$ som optimum

c) $x_1^*(-2y_1^* - y_2^*) = 0$

$x_2^*(-3y_1^* - 2y_2^*) = 0$ $\Rightarrow x_1^*, x_2^* = 0$

$x_3^*(-4y_1^* - 3y_2^*) = 0 \Rightarrow$ inget

$x_4^*(2 + y_1^*) = 0 \Rightarrow$ inget

$x_5^*(5 + y_2^*) = 0 \quad x_5^* = 0$

$y_1^*(2x_1^* + 3x_2^* + 4x_3^* - x_4^* - 2) =$

$= -2(0 + 0 + 4x_3^* - x_4^* - 2) = 0$

$\underline{x_4^* = -2}$

$$y_2^*(x_1 + 2x_2 + 3x_3 - x_5 - 3) =$$

$$= \frac{8}{3}(0 + 0 + 3x_3 - 0 - 3) = 0$$

$$\Leftrightarrow \underline{x_3^* = 1}$$

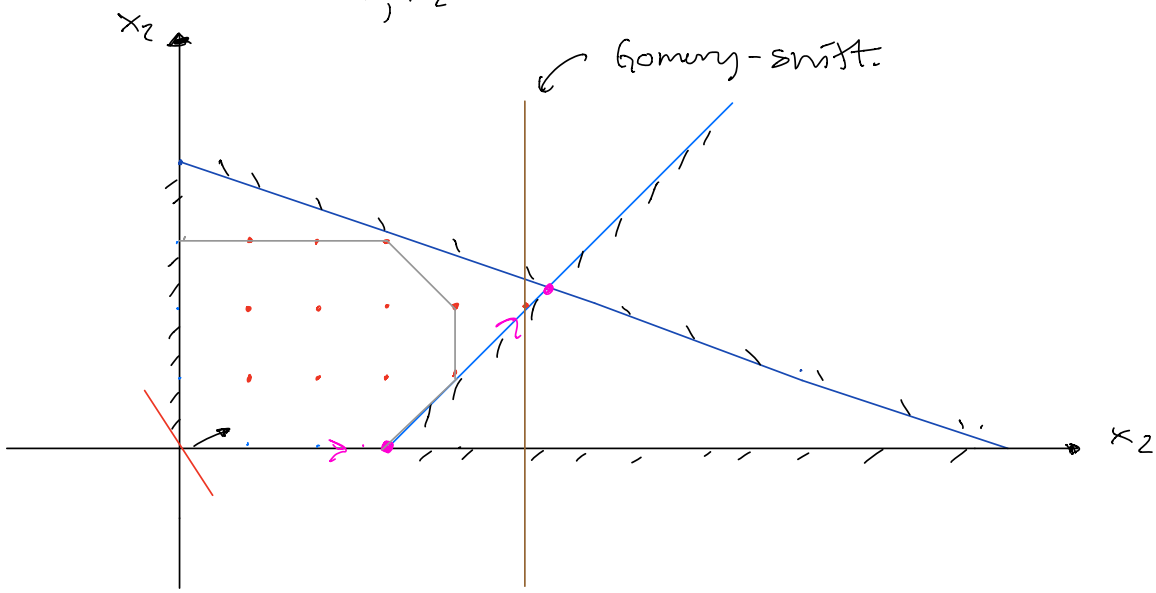
$$\Rightarrow \underline{\underline{2x_4^* + 5x_5^* = 4}}$$

(3) $\max z = 3x_1 + x_2$

då $x_1 - x_2 \leq 3$

$x_1 + 3x_2 \leq 13$

$x_1, x_2 \geq 0$



Infor slackvariabler:

$\max z = 3x_1 + x_2$

då $x_1 - x_2 + s_1 = 3$

$x_1 + 3x_2 + s_2 = 13$

$x_1, x_2, s_1, s_2 \geq 0$

bas	z	x_1	x_2	s_1	s_2	värde
z	1	-3	-1	0	0	0
→ s_1	0	1	-1	1	0	3
s_2	0	1	3	0	1	13

3

-1

Inkommande: x_1

Utgående s_1

bas	z	x_1	x_2	s_1	s_2	värde
z	1	0	-4	3	0	9
x_1	0	1	-1	1	0	3
→ s_2	0	0	4	-1	1	10

1 1/4

$\frac{3}{1} = \frac{12}{4}$

$\frac{10}{4}$

Ingående: x_2

Utgående: s_2

bas	z	x_1	x_2	s_1	s_2	värde
z	1	0	0	2	1	19
→ x_1	0	1	0	3/4	1/4	11/2
x_2	0	0	1	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{5}{2}$

$\underline{\underline{z^* = 19}}$, $x_1^* = 11/2$, $x_2^* = 5/2$

b) $\frac{11}{2} - \lfloor \frac{11}{2} \rfloor = \frac{11}{2} - 5 = \frac{1}{2}$

$\frac{5}{2} - \lfloor \frac{5}{2} \rfloor = \frac{1}{2}$

x_2-raden:

$$x_2 - \frac{1}{4} s_1 + \frac{1}{4} s_2 = \frac{5}{2}$$

Heltalssnitt:

$$x_2 + \lfloor -\tfrac{1}{4} \rfloor s_1 + \lfloor \tfrac{1}{4} \rfloor s_2 \le \lfloor \tfrac{5}{2} \rfloor \Leftrightarrow x_2 - s_1 \le 2$$

$$\Leftrightarrow x_2 - (3 - x_1 + x_2) \le 2 \Leftrightarrow x_1 - 3 \le 2 \Leftrightarrow \underline{\underline{x_1 \le 5}}$$

Ingen fusett då ej tangerar konvexa höljet.

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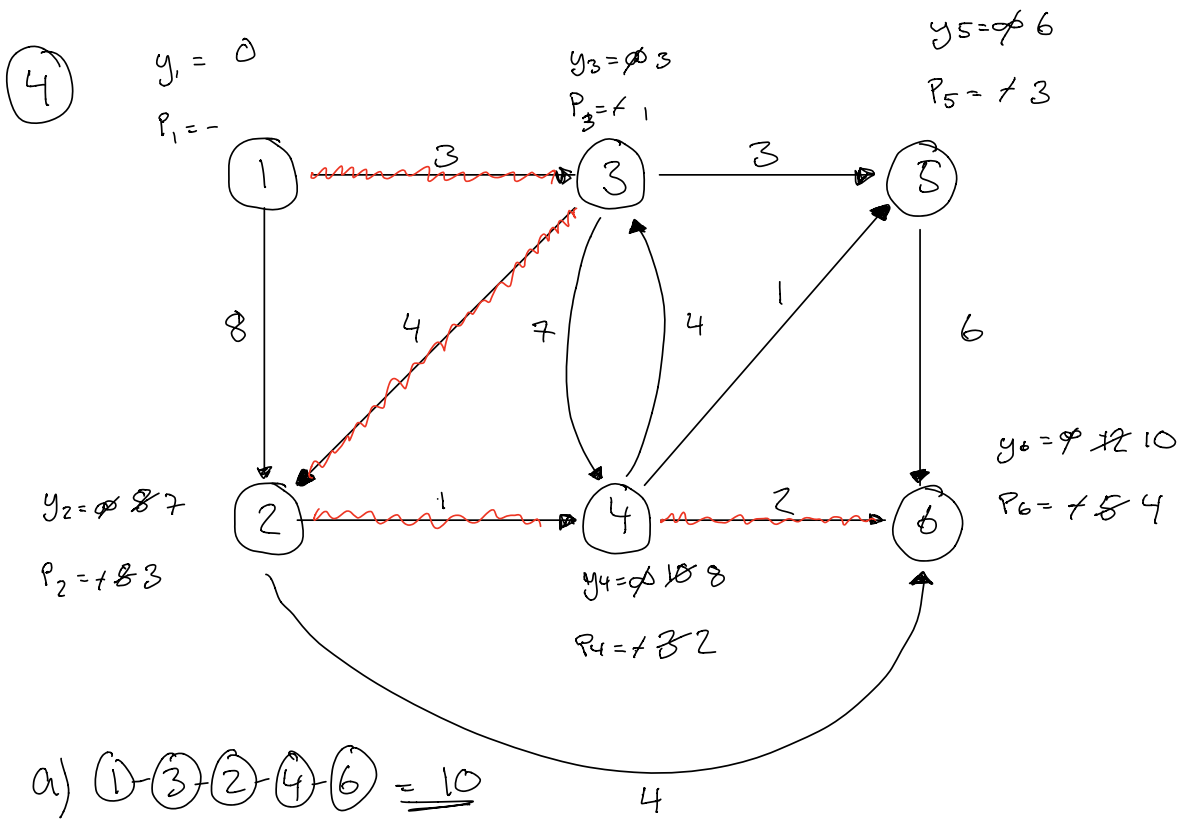

a) 1-3-2-4-6 $= \underline{\underline{10}}$

b) $y_1 = 0$

$y_2 = \min\{y_1 + 8,\ y_3 + 4\} = \min\{8,\ 7\} = 7$

$y_3 = \min\{y_1 + 3,\ y_4 + 4\} = \min\{3,\ 12\} = 3$

$y_4 = \min\{y_2 + 1,\ y_3 + 7\} = \min\{8,\ 10\} = 8$

$y_5 = \min\{y_3 + 3,\ y_4 + 1\} = \min\{6,\ 9\} = 6$

$y_6 = \min\{y_4 + 2,\ y_2 + 4,\ y_5 + 6\} = \min\{10, 11, 12\} = 10$

c)

(5) $\min_{x \in \mathbb{R}^2} f(x) = (x_1-2)^4 + (x_1-2)^2 x_2^2 + (x_2+1)^2$

- $\nabla f = \left(4(x_1-2)^3 + 2(x_1-2)x_2^2 \;,\; 2(x_1-2)^2 x_2 + 2(x_2+1) \right]$
- $\nabla f(1,1) = -(-4-2 \;,\; 2+4)^T = -(-6, 6)^T = \begin{pmatrix} 6 \\ -6 \end{pmatrix} // \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$d^0 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \Rightarrow \underline{x(t) = x^0 + t d^0} = \underline{\begin{pmatrix} 1+t \\ 1-t \end{pmatrix}}, \; t \geq 0$

$\Rightarrow x^1 = \begin{pmatrix} 1+t \\ 1-t \end{pmatrix} = \begin{pmatrix} 5/2 \\ -1/2 \end{pmatrix}$ då $t = 3/2$

Undersøker om konvekt

$\varphi(t) = f(x^t) = \ldots\ldots$

$\varphi''(t) > 0 \Rightarrow$ konvekt

b) $d^k = -\nabla^2 f(x^k)^{-1} \nabla f(x^k)$

$f'x = 4(x_1-2)^3 + 2(x_1-2)x_2^2$

$f''x_1x_1 = 12(x_1-2)^2 + 2x_2^2 \Rightarrow 12 + 2 = 14$

$f''x_1x_2 = 4(x_1-2)x_2 \Rightarrow -4$

$f'x_2 = 2(x_1-2)^2 x_2 + 2(x_2+1)$

$f''x_2x_2 = 2(x_1-2)^2 + 2 \Rightarrow 4$

$\nabla^2 f(x^k) = \begin{pmatrix} 14 & -4 \\ -4 & 4 \end{pmatrix}$

$\Rightarrow \nabla^2 f^{-1} = \frac{1}{56-16} \begin{pmatrix} 4 & 4 \\ 4 & 14 \end{pmatrix} = \frac{1}{40} \begin{pmatrix} 4 & 4 \\ 4 & 14 \end{pmatrix}$

$$\Rightarrow d^u = -\frac{1}{40}\begin{pmatrix} 4 & 4 \\ 4 & 14 \end{pmatrix}\begin{pmatrix} -6 \\ 6 \end{pmatrix} = -\frac{1}{40}\begin{pmatrix} 0 \\ 60 \end{pmatrix} = \begin{pmatrix} 0 \\ -3/2 \end{pmatrix}$$

$$-24 + 84 \qquad \frac{6}{4} = \frac{3}{2}$$

$$d^u = \bar{x}^u - x^u = d^h + x^u = \bar{x}^u$$

$$\Leftrightarrow \begin{pmatrix} 0 \\ -3/2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1/2 \end{pmatrix}$$

$$z^* = \min\ 21x_1 + 26x_2 + 29x_3 + 30x_4 + 31x_5 + 35x_6$$

$$\text{dá} \quad 10x_1 + 8x_2 + 7x_3 + 9x_4 + 7x_5 + 6x_6 \geq 24$$

$$x_1 + x_2 + x_3 = 2$$

$$x_4 + x_5 + x_6 = 1$$

$$x_1, \ldots, x_6 = 0/1$$

$$\Rightarrow h(v) = \min\ 21x_1 + 26x_2 + 29x_3 + 30x_4 + 31x_5 + 35x_6$$

$$+ v(10x_1 + 8x_2 + 7x_3 + 9x_4 + 7x_5 + 6x_6 - 24)$$

$$\text{dá} \quad x_1 + x_2 + x_3 = 2$$

$$x_4 + x_5 + x_6 = 1$$

$$x_1, \ldots, x_6 = 0/1, \quad v \geq 0$$

$$h^* = \max_{v \geq 0} h(v)$$

$$h(1) = -24 + \min\ \overbrace{31x_1 + 34x_2 + 36x_3}^{2\text{st}} + \overbrace{39x_4 + 38x_5 + 41x_6}^{1\text{st}}$$

$$\text{dá} \quad x_1 + x_2 + x_3 = 2$$

$$x_4 + x_5 + x_6 = 1$$

$$x_1, \ldots, x_6 = 0/1,$$

$\Rightarrow \quad h^* = -24 + 31 + 34 + 38 = \underline{79} \quad$ då $\quad x(1,1,0,0,1,0)$

$x(1,1,0,0,1,0) \Rightarrow 10+8 \quad +7 \not\leq 24$ Villkoret ej uppfyllt.

$$h(3) = -72 + \min\ 51x_1 + 50x_2 + 50x_3 + 57x_4 + 52x_5 + 53x_6$$

$$\text{då} \quad x_1 + x_2 + x_3 = 2$$

$$x_4 + x_5 + x_6 = 1$$

$\Rightarrow h^* = -72 + 100 + 52 = 80 \Rightarrow x\underline{(0,1,1,0,1,0)} \Rightarrow$

$\Rightarrow \quad 8 + 7 + 7 \leq 24$ OK!

$\Rightarrow z^* = 26 + 29 + 31 = 86$

$$\left.\begin{array}{l} z^* \geq 79, \text{ ej OK} \\ z^* \geq 80 \\ z^* \leq 86 \end{array}\right\} \Rightarrow 80 \leq z^* \leq 86$$