

# Lösningsgång

TAOP07 – Optimeringslära grundkurs

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①

a) Råmaterial:  $A, B, C$

Viutprocent:  $C_A, C_B, C_C \in \mathbb{R}$

$$C_A \cdot x_A + C_B \cdot x_B + C_C \cdot x_C \in \mathbb{R} \quad (x_A + x_B + x_C)$$

b)  $\min z = \dots + y_1 C_1 + y_2 C_2 + y_3 C_3$

då

$$a^T x \leq b + (y_1 \delta_1 + y_2 \delta_2 + y_3 \delta_3)$$

$$y_1 + y_2 + y_3 \leq 1$$

$$y_i = 0/1, \quad i = 1, 2, 3$$

$$y_i = \begin{cases} 1, & \text{om ska användas, } i = 1, 2, 3 \\ 0, & \text{annars} \end{cases}$$

c)  $y_i a_i^T x = b_i, i = 1, \dots, m$   ~~$b_i$~~   $b_i - M(1-y_i) \leq a_i^T x \leq b_i + M(1-y_i)$

$$y_i = \begin{cases} 1, & \text{om uppfyllas} \\ 0, & \text{annars} \end{cases}$$

$$\sum_{i=1}^m y_i \geq k$$

$$y_i = 0/1$$

$$\textcircled{2} \max z = 4x_1 + 2x_2 + 2x_3$$

$$x_1 - x_2 + x_3 \leq 2$$

$$2x_1 + x_2 + x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0$$

$$\Rightarrow \max z = 4x_1 + 2x_2 + 2x_3$$

$$x_1 - x_2 + x_3 + s_1 = 2$$

$$2x_1 + x_2 + x_3 + s_2 = 10$$

$$x_1, x_2, x_3 \geq 0 \quad s_1, s_2 \geq 0$$

Tablå

bas	z	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	s <sub>1</sub>	s <sub>2</sub>	värde
z	1	-4	-2	-2	0	0	0
s <sub>1</sub>	0	1	-1	1	1	0	2
s <sub>2</sub>	0	2	1	1	0	1	10

↑ 4   ↓ -2

x<sub>1</sub> inkommande, s<sub>1</sub> utgående

bas	z	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	s <sub>1</sub>	s <sub>2</sub>	värde
z	1	0	-6	2	4	0	8
x <sub>1</sub>	0	1	-1	1	1	0	2
s <sub>2</sub>	0	0	3	-1	-2	1	6

Inkommande x<sub>2</sub>, utgående s<sub>2</sub>

bas	z	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	s <sub>1</sub>	s <sub>2</sub>	värde
z	1	0	0	0	0	2	20
x <sub>1</sub>	0	1	0	2/3	1/3	1/3	4
x <sub>2</sub>	0	0	3	-1	-2	1	6

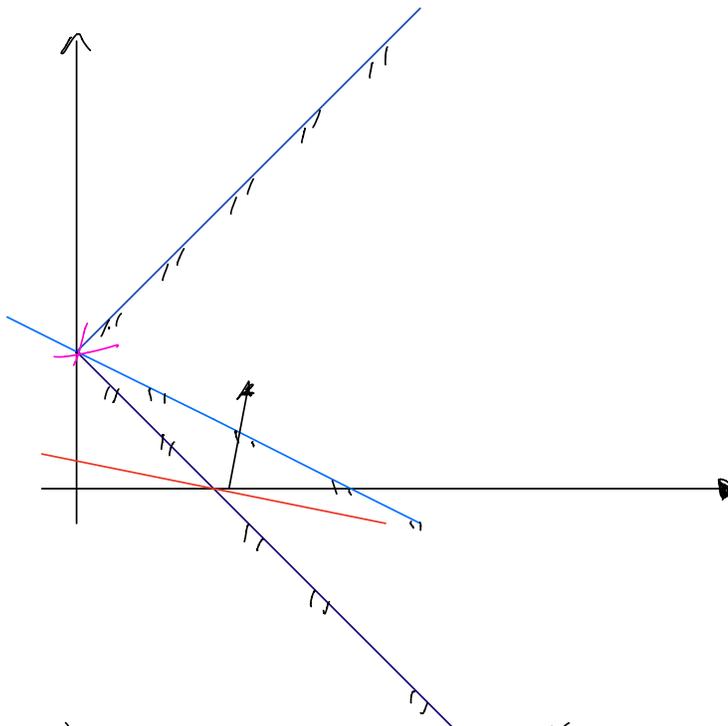
⇒ 0   0   1   -1/2   -2/3   1/3 = 2

$$\Rightarrow \underline{z^* = 20} \quad x^* = (4, 2, 0)^T$$

b)  $\min w = 2y_1 + 10y_2$

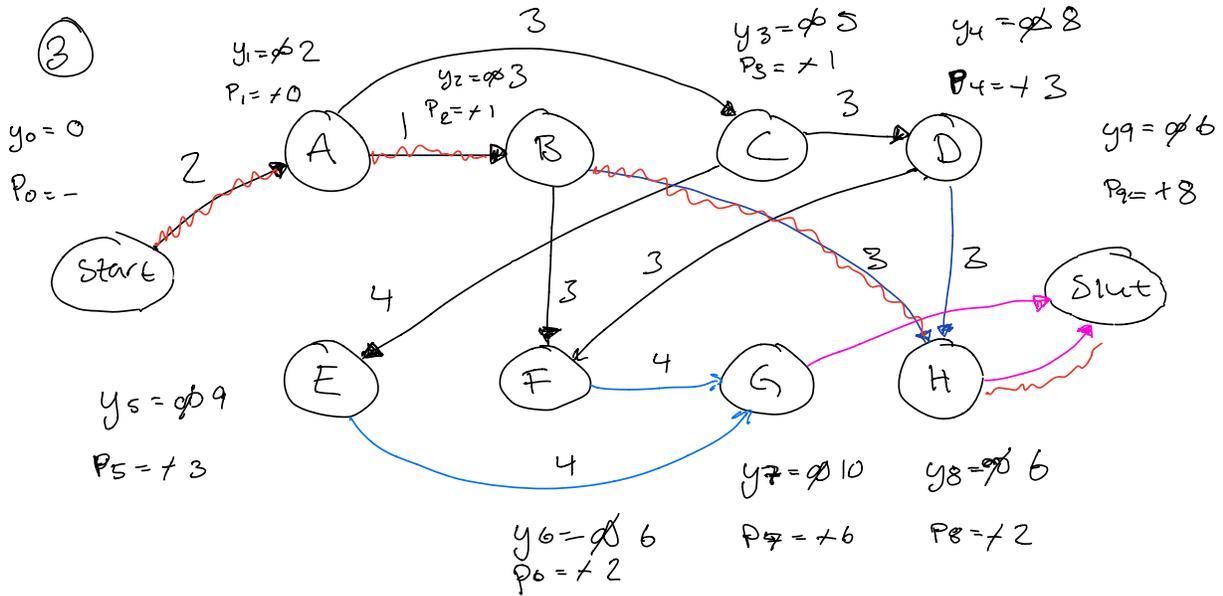
$\textcircled{D}$  Defo  $\begin{cases} y_1 + 2y_2 \geq 4 \\ -y_1 + y_2 \geq 2 \\ y_1 + y_2 \geq 2 \end{cases} \Rightarrow \begin{cases} y_2 \geq 2 - \frac{y_1}{2} & (1) \\ y_2 \geq 2 + y_1 & (2) \\ y_2 \geq 2 - y_1 & (3) \end{cases}$

$y_1, y_2 \geq 0$



$(0, 2) \Rightarrow \text{Optimum} \Rightarrow \underline{\underline{w^* = 20}}$

c)  $\bar{x} \Rightarrow z^* = 20$   
 finiten? JA!



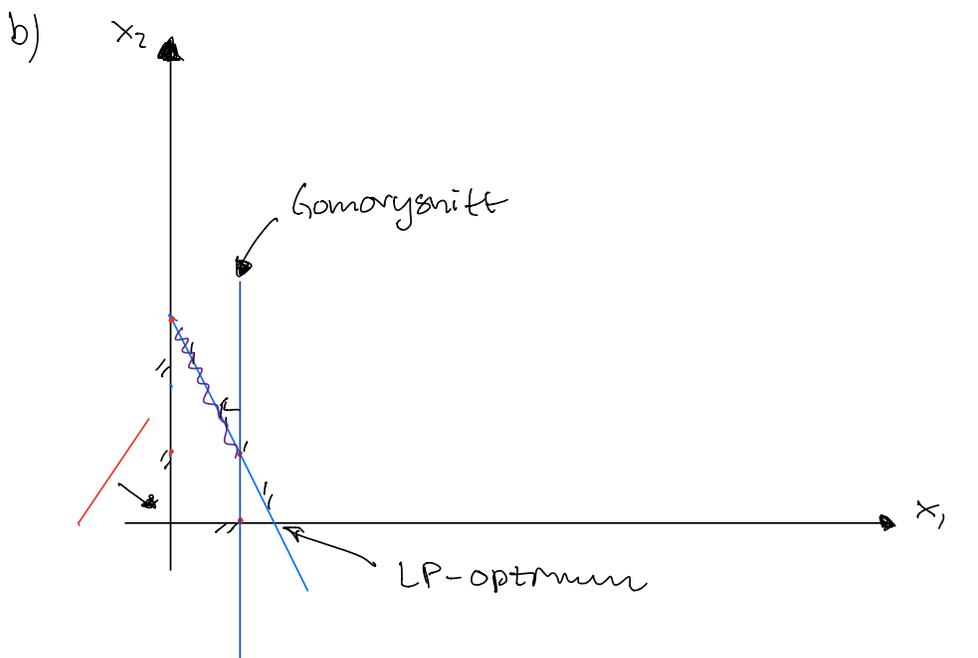
BV: Start  $\rightarrow$  1  $\rightarrow$  2  $\rightarrow$  3  $\rightarrow$  slut  $\Rightarrow$  6  
 A B H

b) Eftersom  $y_7$  har  $y_6$  som föregångare måste man minska tidsåtgången med 5 ty  $10 - 5 = 6$

④  $\max z = 2x_1 - x_2$   
 da  $2x_1 + x_2 \leq 3$   
 $x_1, x_2 \geq 0$ , heilbar

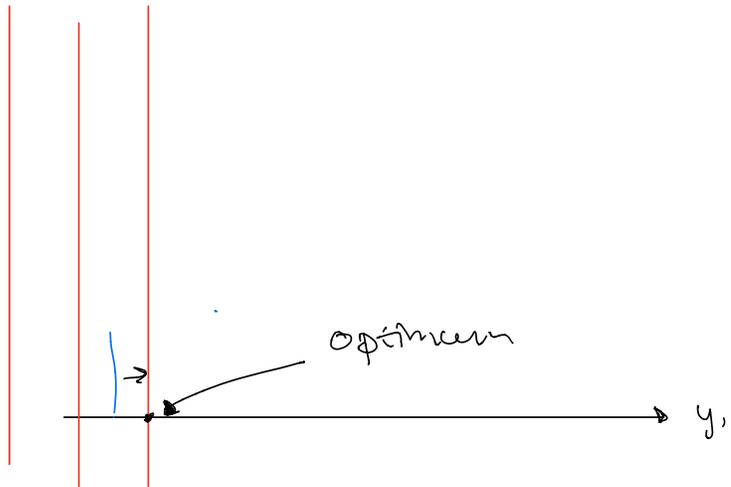
a)  $x_1$  raden:

$$x_1 + \left[\frac{1}{2}\right]x_2 + \left[\frac{1}{2}\right]s_1 \leq \left[\frac{3}{2}\right] \Leftrightarrow x_1 \leq 1$$



Richtig! G-schnitt  $\rightarrow$  wir sind dort optimum!

c)  $\min z = 3y_1$   
 $2y_1 \geq 2 \Rightarrow y_1 \geq 1$   
 $y_1 \geq -1 \quad y_1 \geq -1$   
 $y_1 \geq 0$

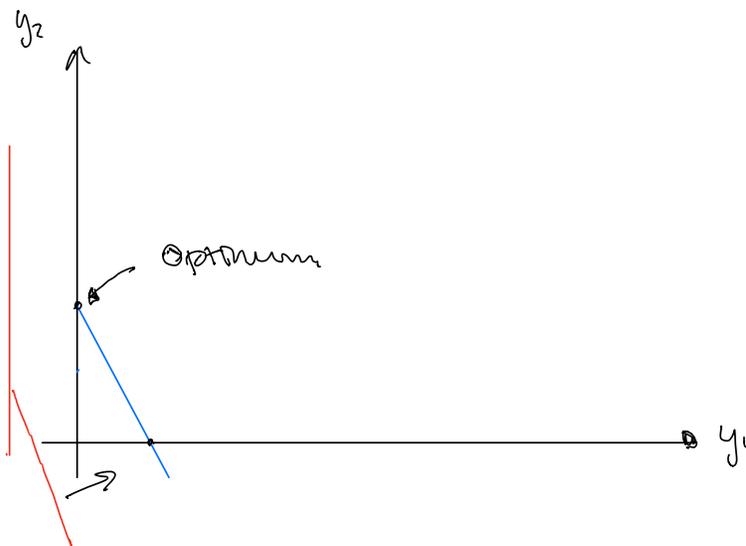


$$\min w = 3y_1 + y_2$$

$$2y_1 + y_2 \geq 2 \Leftrightarrow y_2 \geq 2 - 2y_1$$

$$y_1 \geq -1$$

$$y_1, y_2 \geq 0$$



$$\textcircled{5} \quad f(x) = (k+1)x_1^2 - 3kx_1x_2 + (k+1)x_2^2$$

$$\nabla f(x) = (2(k+1)x_1 - 3kx_2, -3kx_1 + 2(k+1)x_2)$$

$$f''_{x_1x_1} = 2(k+1) \quad f''_{x_1x_2} = -3k$$

$$f''_{x_2x_2} = 2(k+1)$$

$$\Rightarrow \det(\nabla^2 f(x) - I\lambda) = \begin{vmatrix} 2(k+1) - \lambda & -3k \\ -3k & 2(k+1) - \lambda \end{vmatrix} =$$

$$= (2(k+1) - \lambda)(2(k+1) - \lambda) - 9k^2 = 0$$

$$\Leftrightarrow 4(k+1)^2 - 4(k+1)\lambda + \lambda^2 - 9k^2 = 0$$

$$\Leftrightarrow (\lambda - 2(k+1))^2 - 4(k+1)^2 + 4(k+1)^2 - 9k^2 = 0 \Leftrightarrow \lambda = 2(k+1) \pm 3k$$

$\Rightarrow$  positiv + semidefinit da

$$0 \leq 2k+2 \pm 3k \Leftrightarrow -2k \pm 3k \leq 2$$

$$\Leftrightarrow \begin{cases} k \leq 2 \\ k \geq -\frac{2}{5} \end{cases}$$

b)  $k = -1$

$$d\bar{u} = -\nabla^2 f(\bar{x}) \bar{D} f(\bar{x})$$

$$\nabla f(x) = (3kx_2, 3kx_1) \quad -\nabla^2 f(x) = -\begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix}$$

$$\Rightarrow -\begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix}^{-1} = \frac{1}{9} \begin{pmatrix} 0 & -3 \\ -3 & 0 \end{pmatrix}$$

$$\Rightarrow d\bar{u} = \frac{1}{9} \begin{pmatrix} 0 & -3 \\ -3 & 0 \end{pmatrix} \begin{pmatrix} 3 & x_2 \\ 3 & x_1 \end{pmatrix} = \frac{1}{9} (-9x_1, -9x_2) =$$

$$= (-x_1, -x_2)^T$$

$$\nabla f(x)^T d = (3x_2, 3x_1) \begin{pmatrix} -x_1 \\ -x_2 \end{pmatrix} \leq 0 \Leftrightarrow -6x_1x_2 < 0$$

$$\Leftrightarrow x_1x_2 > 0 \Rightarrow x_1, x_2 > 0 \text{ oder } x_1, x_2 < 0$$

$$c) \nabla f(x) = (2(k+1)x_1 - 3kx_2, -3kx_1 + 2(k+1)x_2)$$

$$\bar{d} = \frac{1}{B} \nabla f(x) = \frac{1}{B} \begin{pmatrix} 3kx_2 - 2(k+1)x_1 \\ 3kx_1 - 2(k+1)x_2 \end{pmatrix} \quad B=1$$

$$d \parallel -\nabla f(x)$$

$$\nabla f(x) \cdot \bar{d} = \begin{pmatrix} 2(k+1)x_1 - 3kx_2 \\ -3kx_1 + 2(k+1)x_2 \end{pmatrix}^T \begin{pmatrix} 3kx_2 - 2(k+1)x_1 \\ 3kx_1 - 2(k+1)x_2 \end{pmatrix} =$$

$$= -(2(k+1)x_1 - 3kx_2)^2 - (3kx_1 - 2(k+1)x_2)^2 < 0$$

Gatter dies fur alle  $(x_1, x_2, k)$

$$\textcircled{6} \min f(x)$$

$$\text{d.h. } g_1(x) \leq 0$$

$$g_2(x) \leq 0$$

$$x \in \mathbb{R}$$

$$\Rightarrow L(x, v) \min f(x) + v_1 g_1(x) + v_2 g_2(x)$$

$$\textcircled{P} \quad x \in \mathbb{R}$$

Dualen

$$\max L(x, v)$$

$$v \geq 0$$

Bivillkoren måste vara uppfyllda  $\Rightarrow$

$$\text{både } g_1(x), g_2(x) \leq 0$$

$$\underline{u=4}$$

$$L(x, u) = 12 + \frac{3}{2} \cdot (-1) + 4 \cdot \left(-\frac{1}{2}\right) = 12 - \frac{3}{2} - \frac{4}{2} = \frac{24}{2} - \frac{7}{2} = \frac{17}{2}$$

$$\underline{u=6}$$

$$L(x, u) = \underbrace{12,25}_{48} + \frac{5}{4} \cdot \left(-\frac{1}{4}\right) + \frac{15}{4} \cdot \left(-\frac{1}{4}\right) = 12,25 - \frac{5}{16} - \frac{15}{16} =$$

$$\left/ \frac{48}{4} \cdot \frac{4}{4} = \frac{98}{8} \cdot \frac{2}{2} = \frac{196}{16} \right.$$

$$= \frac{176}{16} = \frac{88}{8} = \frac{44}{4} = \frac{22}{2} = \underline{\underline{11}}$$

$$h(v) = \min \left\{ \frac{17}{2}, u \right\} = \frac{17}{2} \approx 8,5 \leq 12$$