

Uppgifter 6

TAOP07 – Optimeringslära grundkurs

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Lektionsgenomgång

$$\max z = 9x_1 + 8x_2 + 7x_3 + 6x_4$$

da $3x_1 + 2x_2 + 4x_3 + 5x_4 \leq 6$

$$\begin{array}{l} \uparrow \\ 3x_1 \\ \uparrow \\ 3/2 \\ x_i \neq 0 \end{array} \quad 0 \leq x_i \leq 1 \quad \begin{array}{l} \uparrow \\ 3x_4 \\ \uparrow \\ 3x_4 \end{array}$$

$$x_i \in \{0, 1\}$$

$$x_2^* = 1 \Rightarrow 6 - 2 \cdot 1 = 4$$

$$x_1^* = 1 \Rightarrow 4 - 3 \cdot 1 = 1$$

$$\text{ty } \frac{2}{9} > \frac{6}{5} \Rightarrow x_3^* = \frac{1}{4} \Rightarrow 1 - 1 = 0$$

$$x_4^* = 0$$

$$x_1' = 3x_1, \quad x_2' = 2x_2, \quad x_3' = 4x_3, \quad x_4' = 5x_4$$

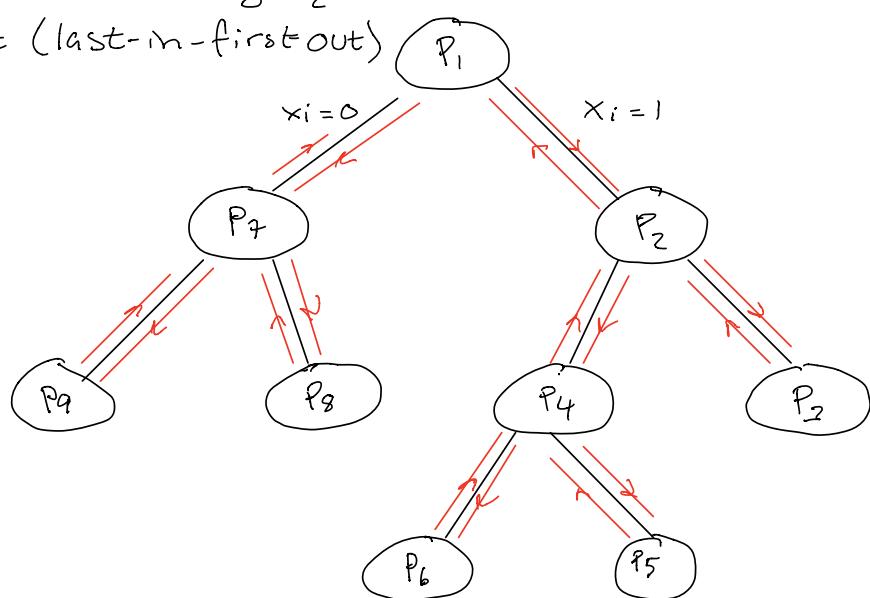
$$\max z = \frac{9}{3}x_1' + \frac{8}{2}x_2' + \frac{7}{4}x_3' + \frac{6}{5}x_4'$$

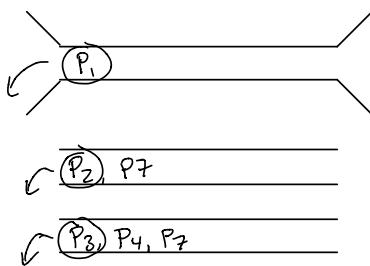
$$x_1' + x_2' + x_3' + x_4' \leq 3$$

↑

ty $\frac{8}{2}x_2'$ är störst.

depth-first (last-in-first-out)





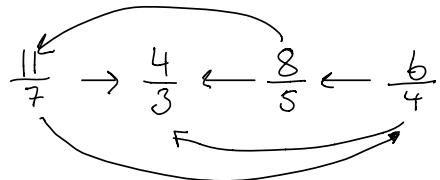
$$\max z = 11x_1 + 4x_2 + 8x_3 + 6x_4$$

$$\text{dai} \quad 7x_1 + 3x_2 + 5x_3 + 4x_4 \leq 14$$

$$x_i \in \{0, 1\}$$

- a) choose $x_i=1$ to branch 1st
- b) Follow the depth-first rule

$$x_3 \rightarrow x_1 \rightarrow x_4 \rightarrow x_2$$



$$P_0: x_{LP} = (1, 0, 1, \underline{0.5})^T, z_{LP}=22 \Rightarrow VBD=22$$

$$x_3 = 1 \Rightarrow 14 - 5 = 9$$

$$x_1 = 1 \Rightarrow 9 - 7 = 2$$

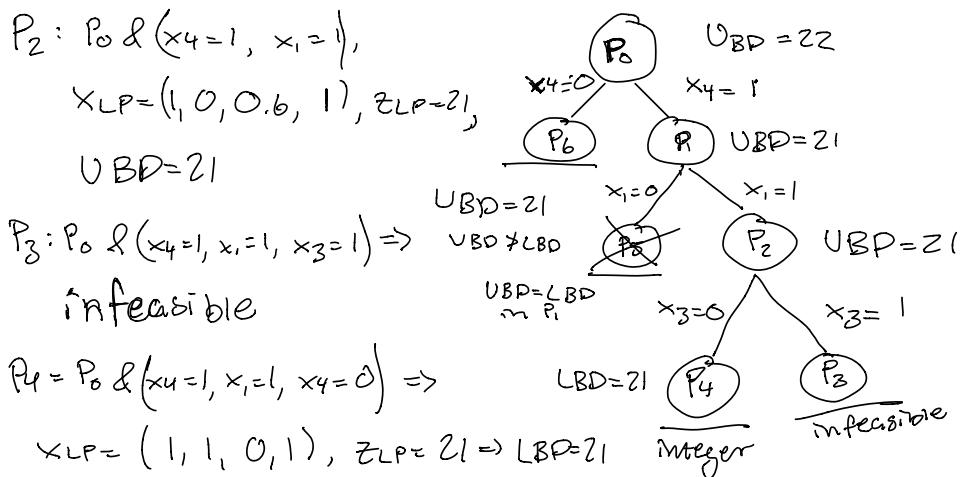
$$x_2 = \frac{2}{4} \Rightarrow 2 - 2 = 0$$

$$x_4 = 0$$

$$P_1: P_0 \text{ d } (x_4=1), x_{LP} = (\underline{0}, 7/4, 0, 1)^T, z_{LP}=21.85 \Rightarrow VBB=21$$

$$x_4 = 1 \Rightarrow 14 - 4 = 10$$

:



$P_5: P_0 \& (x_4=1, x_1=0) \Rightarrow$
 $\Rightarrow \text{no reason to solve because } UBD = LBD$

$P_6: P_0 \& (x_4=0) \Rightarrow$
 $x_{LP} = (1, 0.667, 1, 0)^T, z_{LP} = 21.65 \Rightarrow UBD = 21$

The optimum solution:

$$x^* = (1, 1, 0, 1)^T$$

$$z^* = 21$$

(89)

$$\text{min } z = 2x_1 + x_2 + 2x_3 + x_4 + x_5$$

$$\text{då } x_1 + x_2 \geq 1$$

$$x_1 + x_2 + x_3 + x_4 + x_5 \geq 1$$

$$+ x_3 + x_4 \geq 1$$

$$x_3 + x_4 + x_5 \geq 1$$

$$x_2 + x_4 + x_5 \geq 1$$

$$x_i \in \{0, 1\}$$

$$(90) \quad \text{min } z = 36x_1 + 70x_2 + 145x_3 + 170x_5 - 50y_1 - 50y_2$$

$$\text{då } x_1 + 2x_2 + 4(x_3 + y_4) + 5 \cdot 15x_5 \geq 90$$

$$15y_1 \leq x_1$$

$$15y_2 \leq x_2$$

$$10y_4 \leq x_4$$

$$(x_3 + x_5 \leq 1) \quad x_4 \leq M(1 - x_5)$$

$$x_1, x_2, x_3, x_5 \geq 0 \text{ och heltal}$$

$$x_5 = \begin{cases} 1 & \text{om alla körps} \\ 0 & \text{annars} \end{cases} \quad y_1, y_2, y_3 = 0/1$$

$$y_1 = \begin{cases} 1 & \text{om räbott på en-meter} \\ 0 & \text{annars} \end{cases}$$

$$y_2 = \text{summa}$$

$$y_4 = \begin{cases} 1 & \text{om gratis 4-meters} \\ 0 & \text{annars} \end{cases}$$

$$(98) \text{ a) } x_1 + x_2 \leq 2 + My_1$$

$$2x_1 + 3x_2 \geq 8 - My_2$$

$$y_i \in \{0, 1\}, \quad M \gg 1$$

$$y_1 + y_2 = 1$$

$$\text{b) } x_3 = 6y_0 + 5y_5 + 9y_9 + 12y_{12}$$

$$y_0 + y_5 + y_9 + y_{12} = 1$$

$$y_0 + y_5 + y_9 + y_{12} \in \{0, 1\}$$

$$\text{c) } x_4 + x_5 \leq 2 + My_1$$

$$x_4 \leq 1 + My_2$$

$$x_5 \leq 5 + My_3$$

$$x_4 + x_5 \geq 3 - My_4$$

$$y_1 + y_2 + y_3 + y_4 \leq 2$$

$$y_1, y_2, y_3, y_4 \in \{0, 1\}$$

(103)

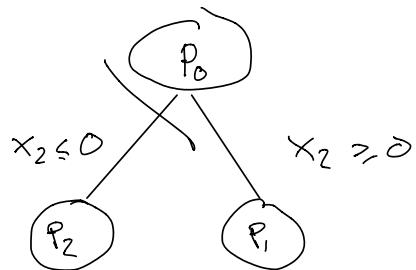
$$\max z = 11x_1 + 6x_2$$

$$\text{da} \quad 2x_1 + 3x_2 \leq 12$$

$$2x_1 + x_2 \leq 7$$

$$x_1 \geq 0, x_2 \geq 0, \text{ hetalesy.}$$

a) $P_0: x_{LP} = (3 \ 0)^T$ $Z_{LP} = 33 \Rightarrow U_{BD} = 33$



P_1 :	bus	-2	x_1	x_2	s_1	s_2	
	-2	1	-11	-6	0	0	33
	x_1	0	2	3	1	0	12
$\rightarrow x_2$	0	2	1	0	1	7	$\frac{11}{2}$

	bus	-2	x_1	x_2	s_1	s_2	
	-2	1	0	$-\frac{1}{2}$	0	$\frac{11}{4}$	$\frac{77}{2}$
	$\rightarrow x_1$	0	0	2	1	-1	5
	x_2	0	1	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{7}{2}$

	bus	-2	x_1	x_2	s_1	s_2	
	-2	1	0	0	$\frac{1}{2}$	$\frac{9}{4}$	(4)
	$\rightarrow x_1$	0	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{5}{2}$
	x_2	0	1	0	0	$\frac{1}{2}$	$-\frac{3}{2}$

