Föreläsning 15

TMME04 – Mekanik II

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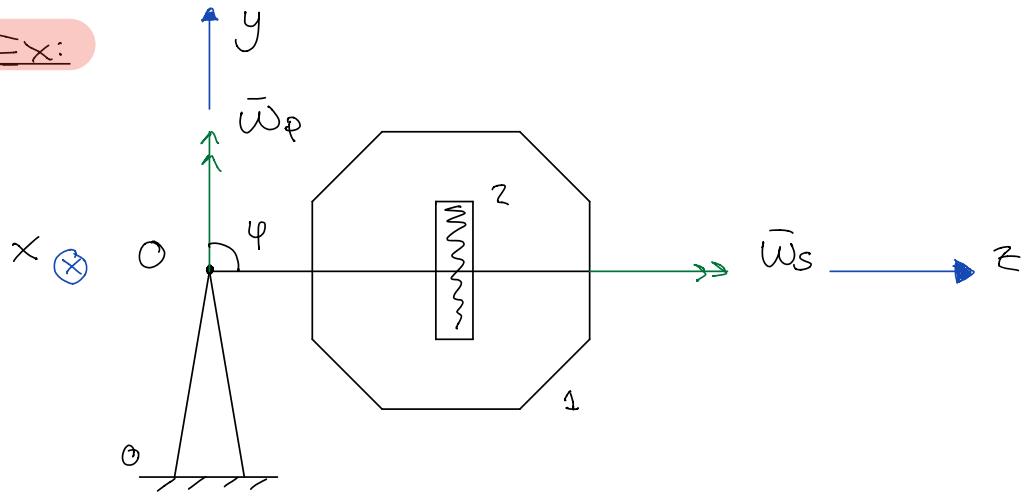
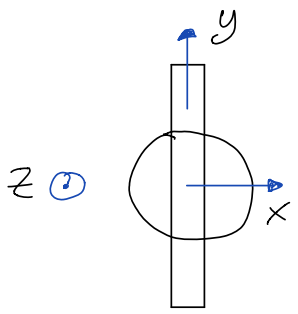
Ex:





Rent precesserande gyrosnurra ($\varphi = 90°$, konst).
Tecknа precessionen $\bar{\omega}_p$ givet spinnet $\bar{\omega}_s$. $\bar{\omega}_s$, $\bar{\omega}_p$ konst.

Studera ytterramen (1) & (2) som ett system

Euler II

$$\bar{M}_O^{ext} = \sum_{j=1}^{2} \dot{\bar{h}}_O^{(j)} = \sum_{j=1}^{2} \left\{ \left(\frac{d\bar{h}_O^{(j)}}{dt} \right)_r + \bar{\omega}_r \times \bar{h}_O^{(j)} \right\}, \quad (1)$$

① fix i i-ram

$$\bar{h}_O^{①} = I_O^{①} \bar{\omega}_1, \quad \bar{h}_O^{②} = \bar{I}_O^{②} \bar{\omega}_2,$$

○ fix i ytterramen och rotorn $\begin{pmatrix} ① \\ ② \end{pmatrix}$

Val av referensram r att derivera i:

$r = 1$

For in $Oxyz$ fixt i ramen.

$$\bar{\omega}_1 = \bar{\omega}_p = \omega_p \hat{y}$$

$$\omega_2 = \bar{\omega}_{2/0} = \bar{\omega}_{2/1} + \bar{\omega}_{1/0} = \omega_s \hat{z} + \omega_p \hat{y}$$

$$\bar{h}_O^{(1)} = \begin{bmatrix} * & 0 & * \\ * & I_{Oyy}^{(1)} & * \\ * & 0 & * \end{bmatrix} \begin{bmatrix} 0 \\ \omega_p \\ 0 \end{bmatrix} = I_{Oyy}^{(1)} \omega_p \hat{y}$$

$$\left(\frac{d\bar{h}_O^{(1)}}{dt} \right)_1 = \bar{0}$$

$$\bar{h}_O^{(2)} = \begin{bmatrix} * & & \\ * & I_{Oyy}^{(2)} & \\ * & & I_{Ozz}^{(2)} \end{bmatrix} \begin{bmatrix} 0 \\ \omega_p \\ \omega_s \end{bmatrix} = I_{Oyy}^{(2)} \omega_p \hat{y} + I_{Ozz}^{(2)} \omega_s \hat{z}$$

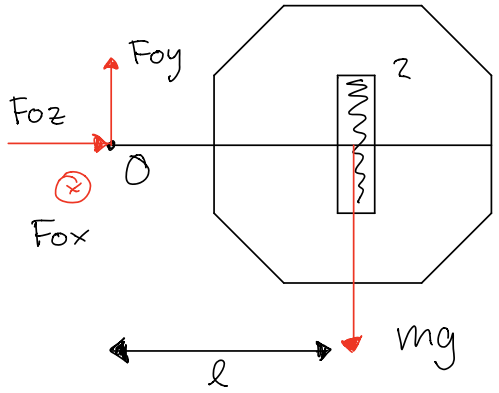
$$\left(\frac{d\bar{h}_O^{(2)}}{dt} \right) = \bar{0}$$

(1) $\Rightarrow$

$$\bar{M}_O^{ext} = \omega_1 \times \left(h_O^{(1)} + h_O^{(2)} \right) = \omega_p \hat{y} \times \left(I_{Oyy}^{(1)} \omega_p \hat{y} + I_{Ozz}^{(2)} \omega_s \hat{z} \right) = I_{Ozz}^{(2)} \left(\omega_p \hat{y} \right) \times \left(\omega_s \hat{z} \right) =$$

$$= I_{Ozz}^{(2)} \omega_p \omega_s \hat{x}. \quad (2)$$

Frilägg:



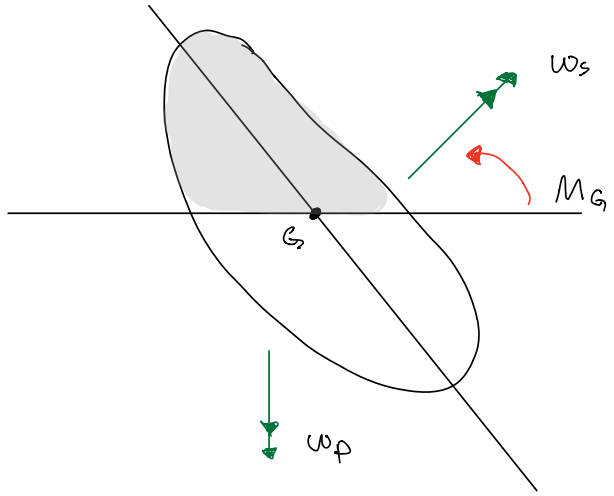
$$\bar{M}_O^{ext} = mg\,l\,\hat{x}$$

$$\therefore\ mg\,l = I_{Ozz}\,\omega_p\,\omega_s \iff \omega_p = \frac{mg\,l}{I_{Ozz}\,\omega_s}, \uparrow$$

(2) ger att

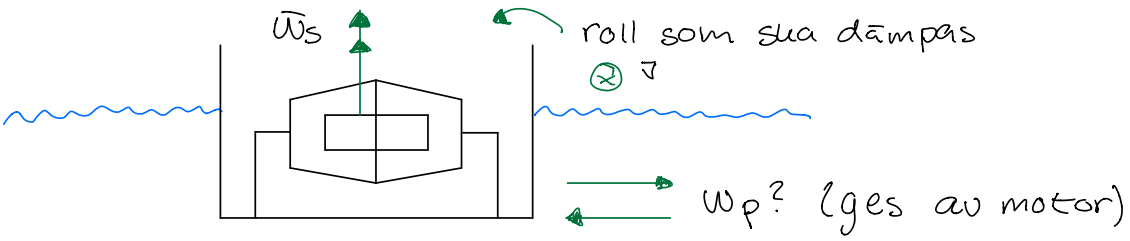
momentet, precessionen och spinnet utgör ett högersystem $\{\bar{M}_O, \bar{\omega}_p, \bar{\omega}_s\}$ ← bra skit

sol.

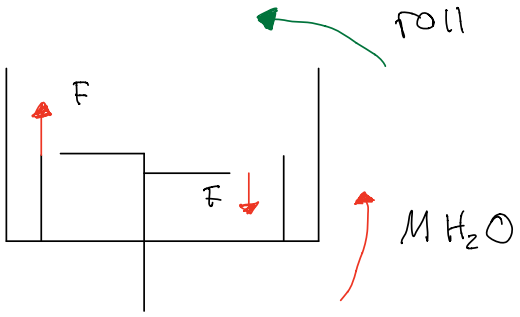


Påtvingas en spinnande rotor en precession, skapas ett moment

Ex: Stabilisering av fartyg

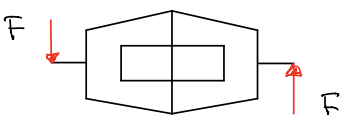


Frilägg fartyget:



ger rolldämpande moment

Frilägg gyrot:



dvs M ↺

$\{\bar{M}, \bar{\omega}_P, \bar{\omega}_S\}$ högersystem $\Rightarrow$ $\bar{\omega}_P$, $\longrightarrow$

Cykel: